Quantum criticality, the AdS/CFT correspondence, and the cuprate superconductors

Talk online: sachdev.physics.harvard.edu

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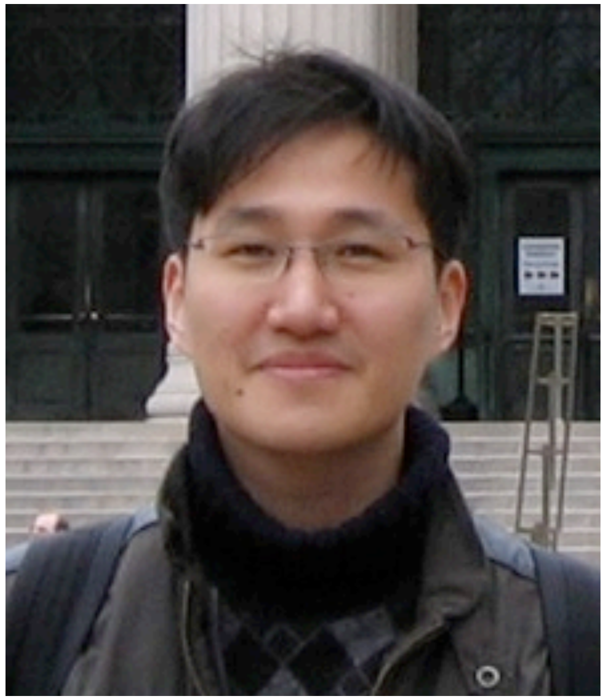
Max Metlitski, Harvard



Eun Gook Moon, Harvard

Frederik Denef, Harvard
Sean Hartnoll, Harvard
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

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Outline

1. The superfluid-insulator transition

   Quantum criticality and
   the AdS/CFT correspondence

2. Graphene

   `Topological' Fermi surface transition

3. The cuprate superconductors

   Fluctuating spin density waves, and
   pairing by "topological" gauge fluctuations

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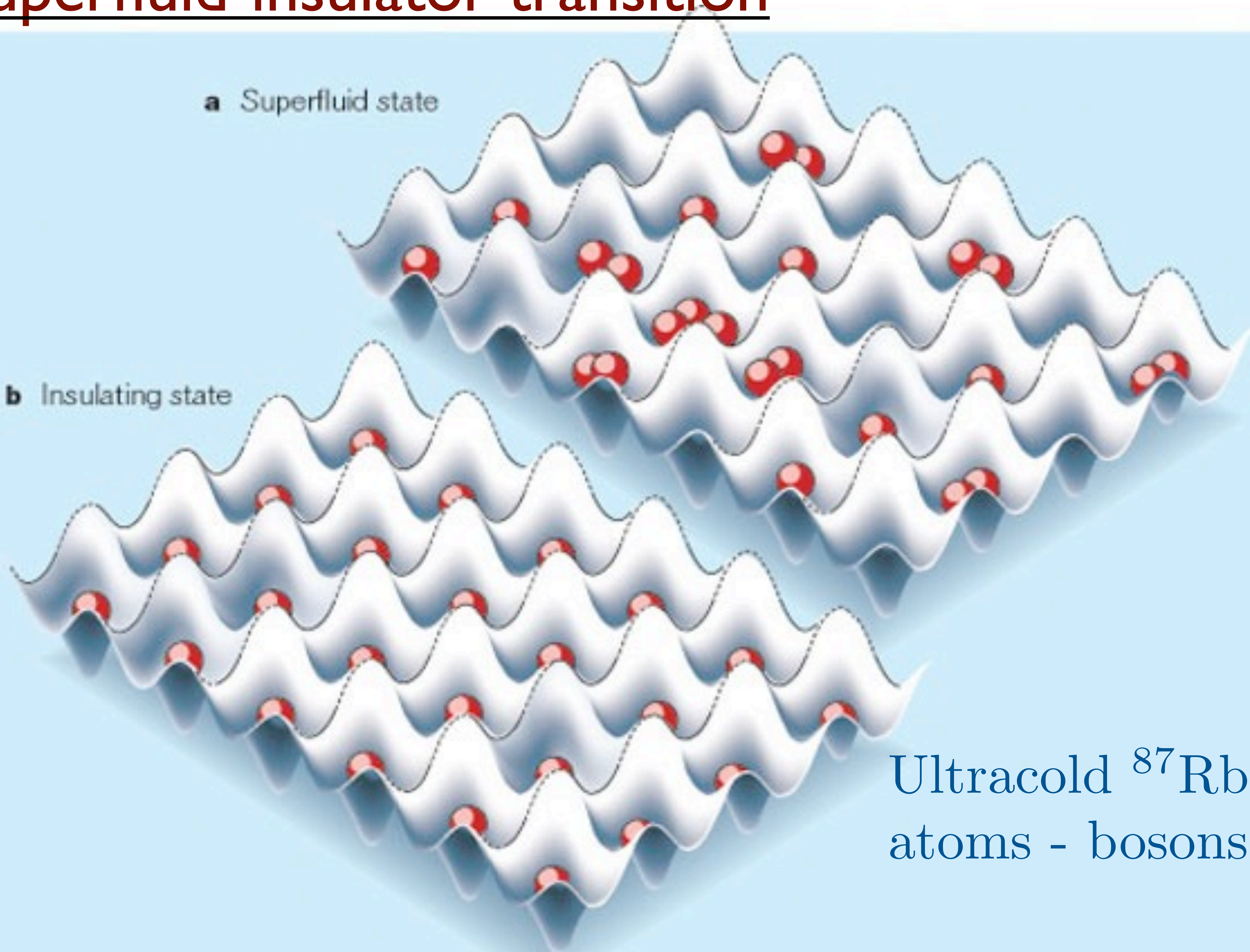
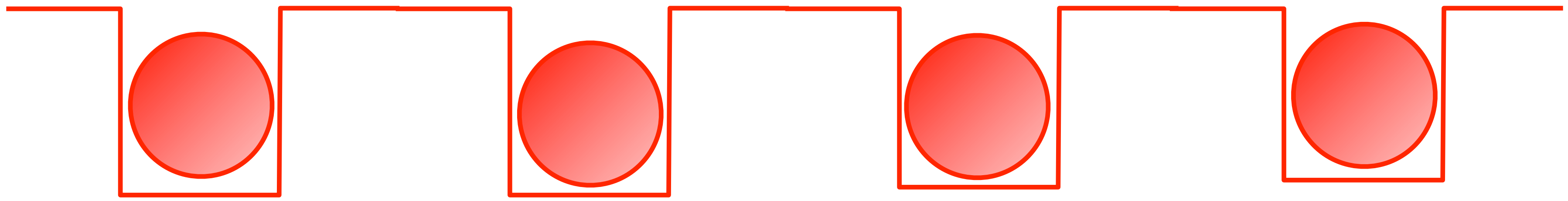
Superfluid-insulator transition



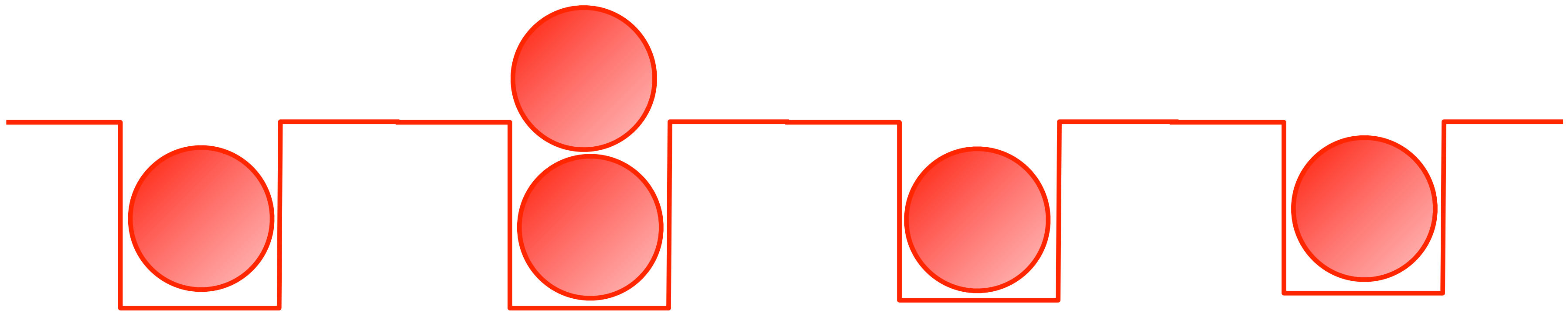
Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



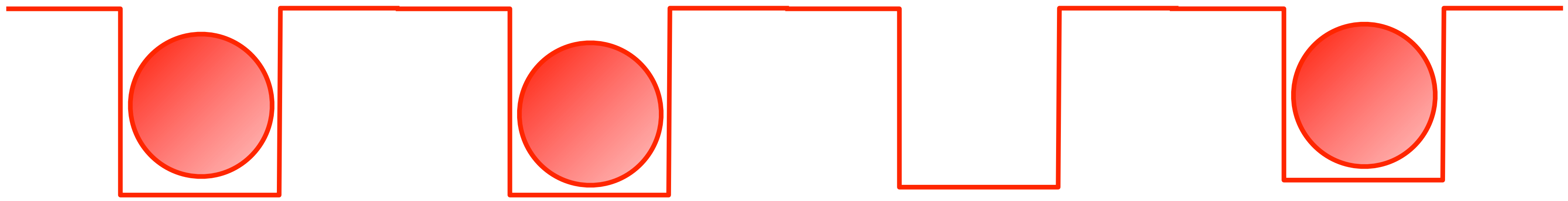
Insulator (the vacuum) at large U

Excitations:



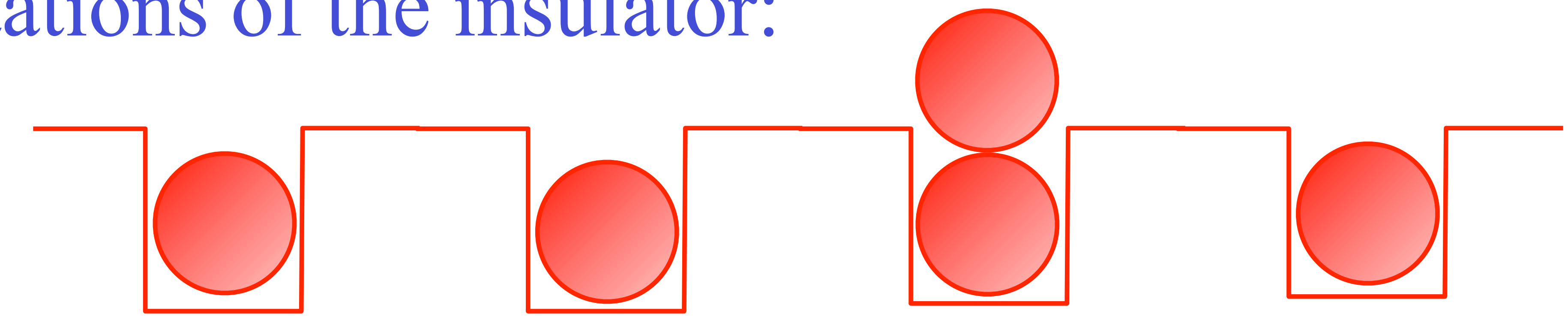
Particles $\sim \psi^\dagger$

Excitations:

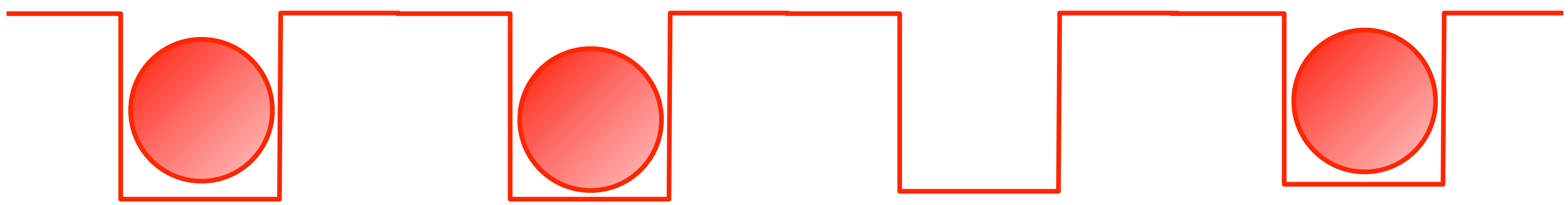


Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$



Holes ~ ψ

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

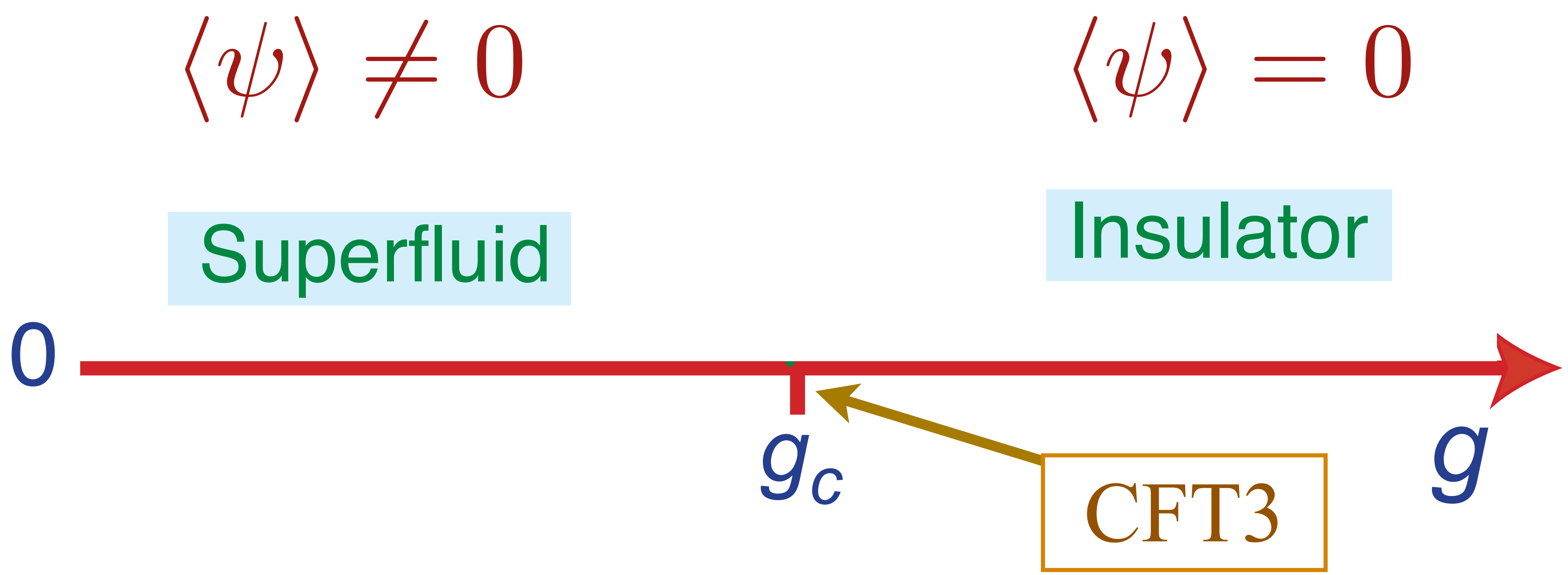
.

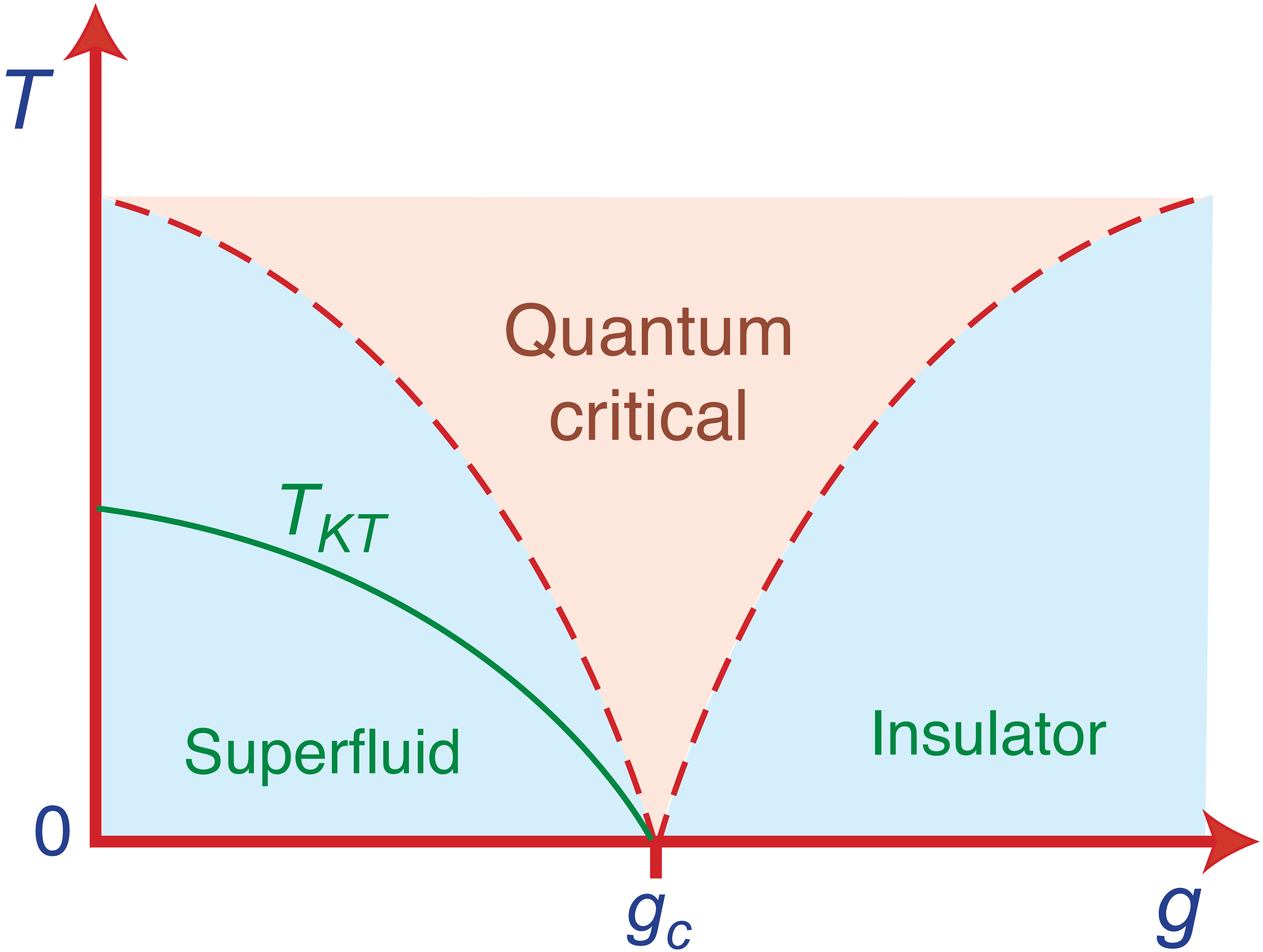
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

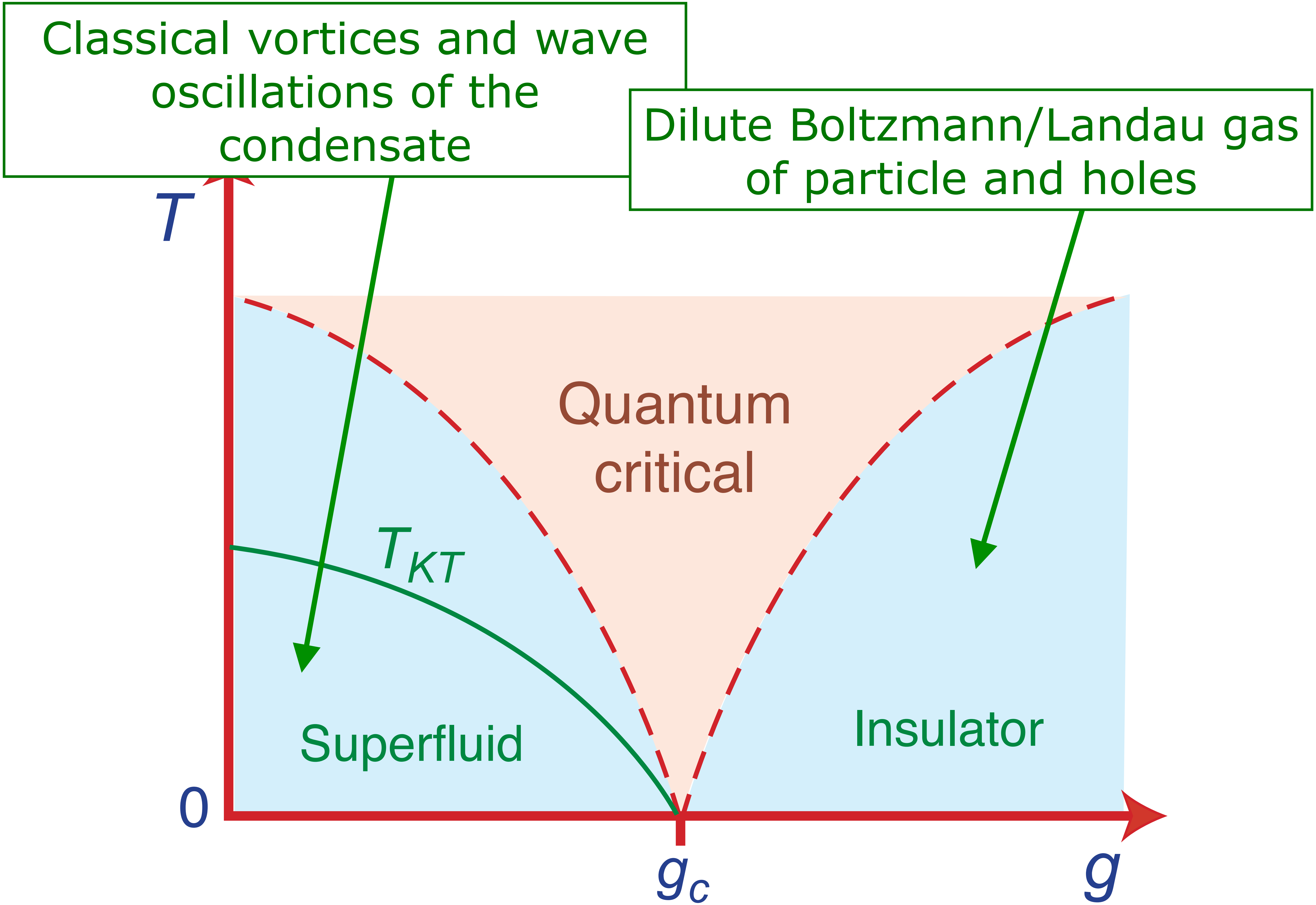
$$\text{Insulator} \quad \Leftrightarrow \quad \langle \psi \rangle = 0$$

$$\text{Superfluid} \quad \Leftrightarrow \quad \langle \psi \rangle \neq 0$$

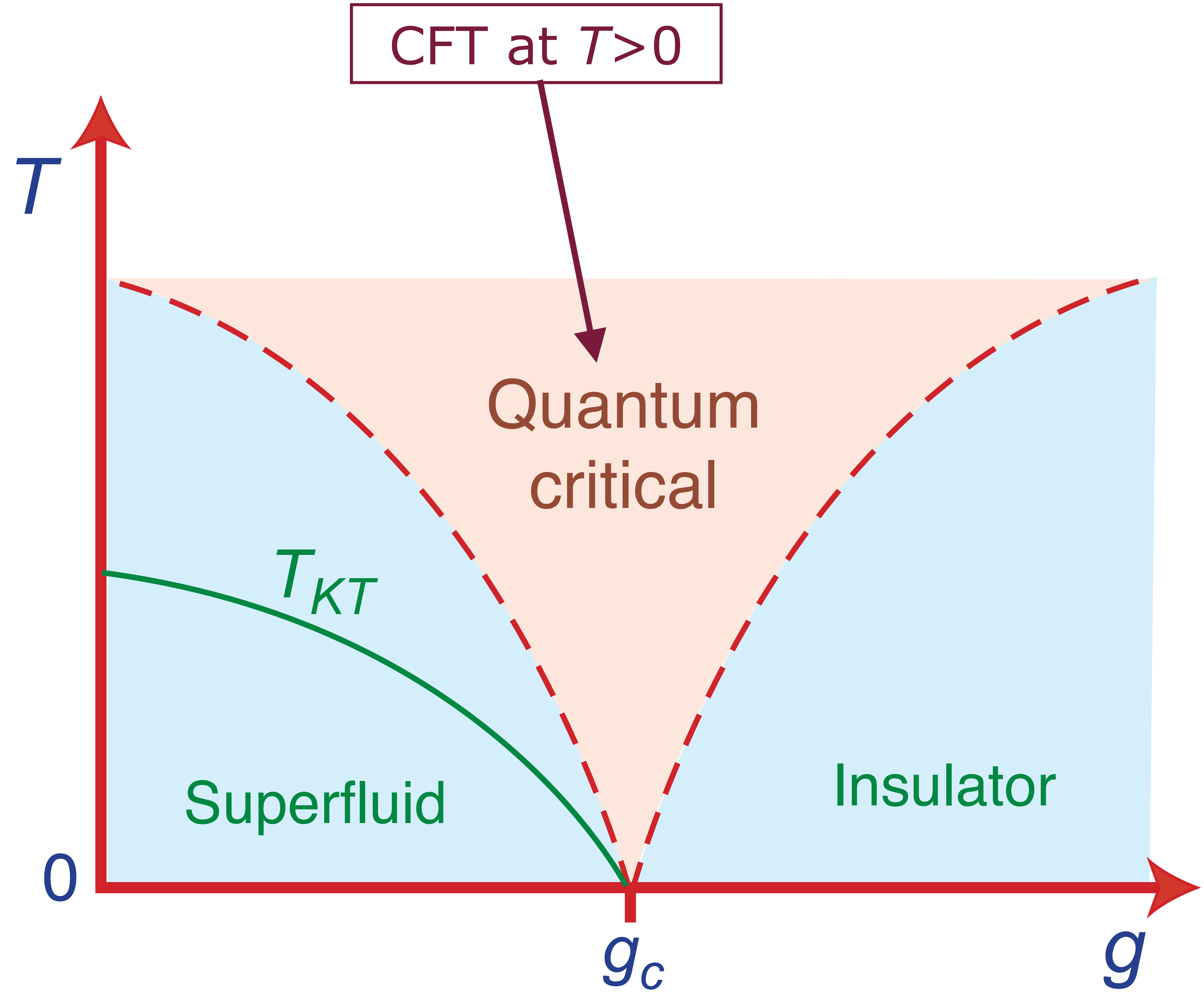
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g



Classical vortices and wave oscillations of the condensate
Dilute Boltzmann/Landau gas of particle and holes
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at $T>0$
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$

$$\sigma_{\text{Insulator}}(T \to 0) = 0$$

$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

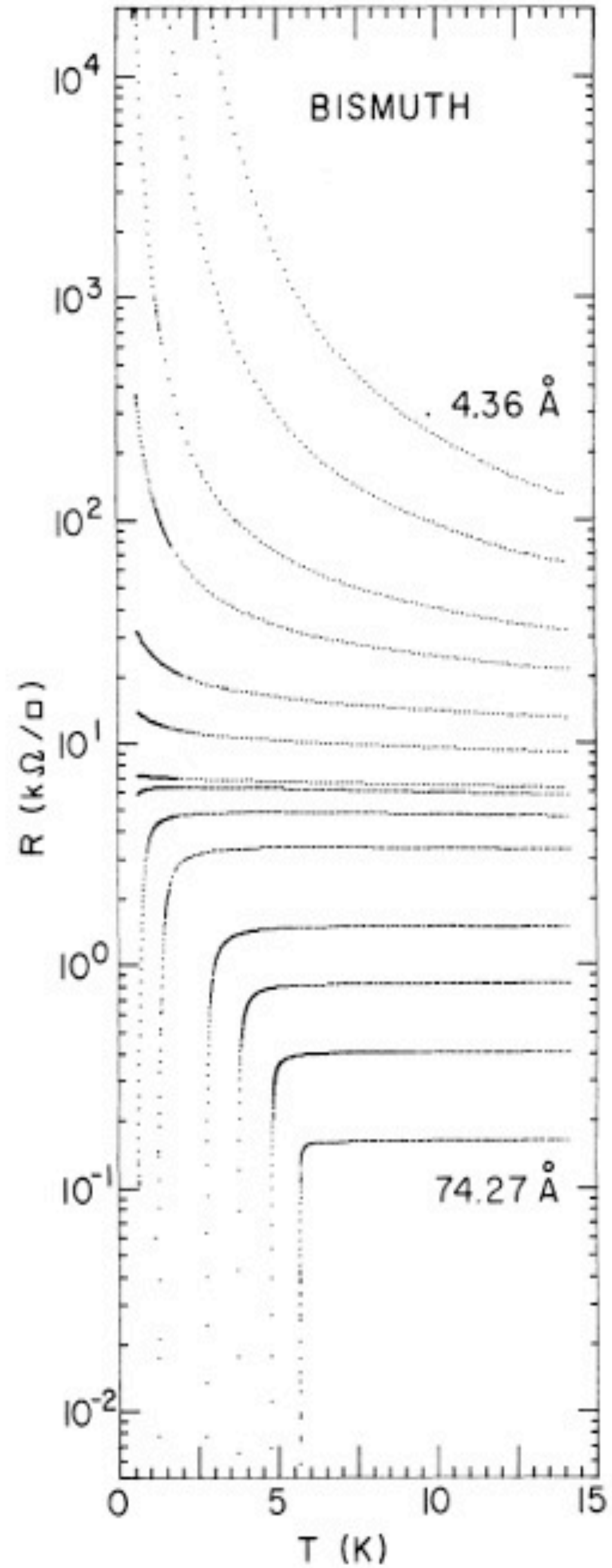


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\boxed{\tau_R \gtrsim \frac{\hbar}{k_B T}}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\boxed{\sigma = \frac{4e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

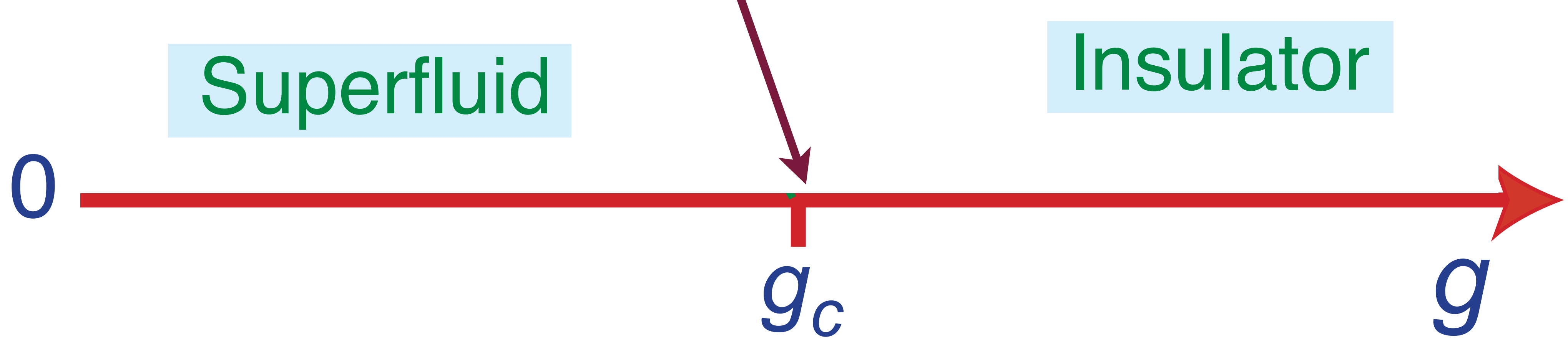
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

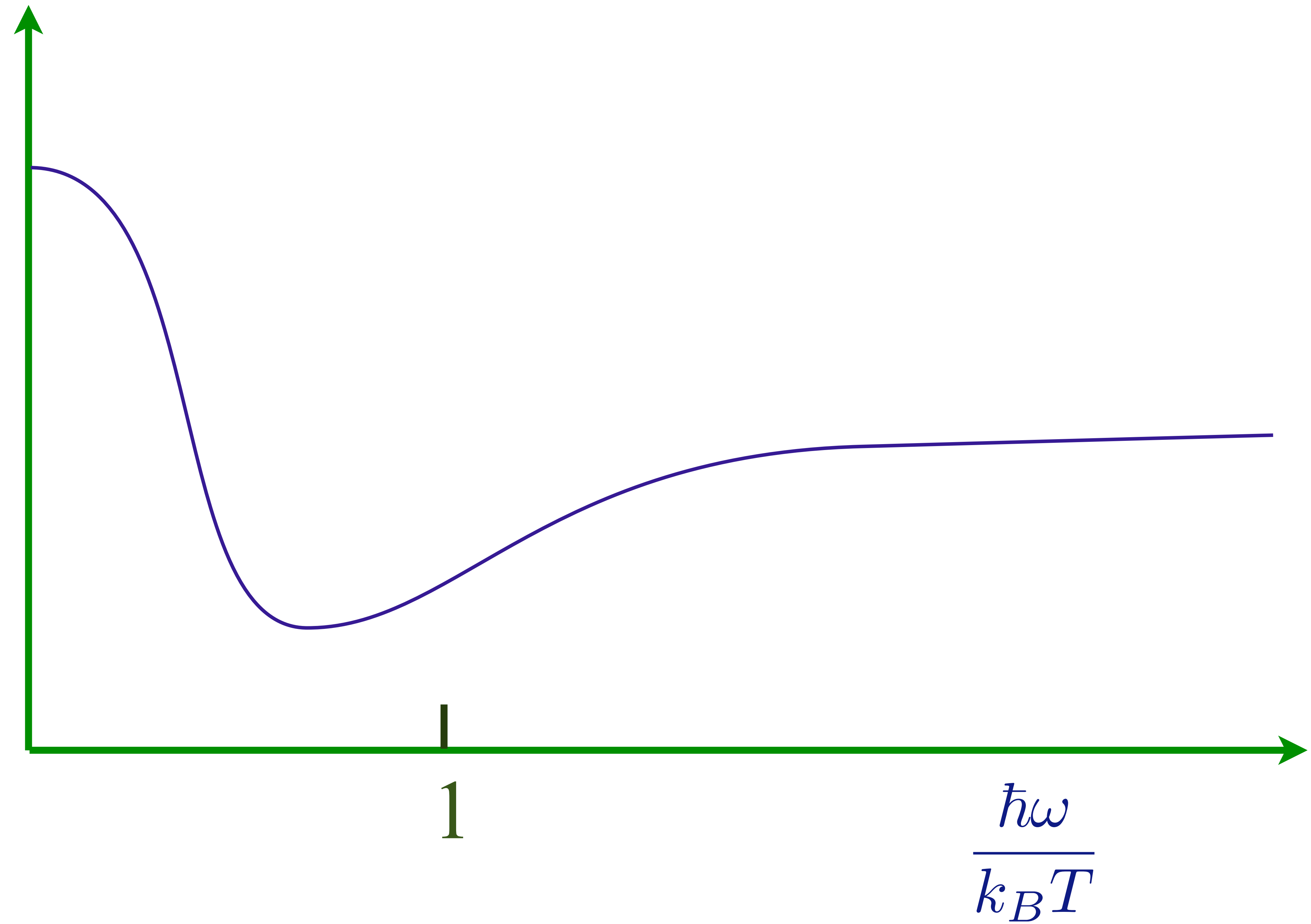
P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

$$\sigma = \frac{4e^2}{h}\Sigma$$

Σ, a universal number.



$$\sigma = \frac{4e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$



K. Damle and S. Sachdev, 1997

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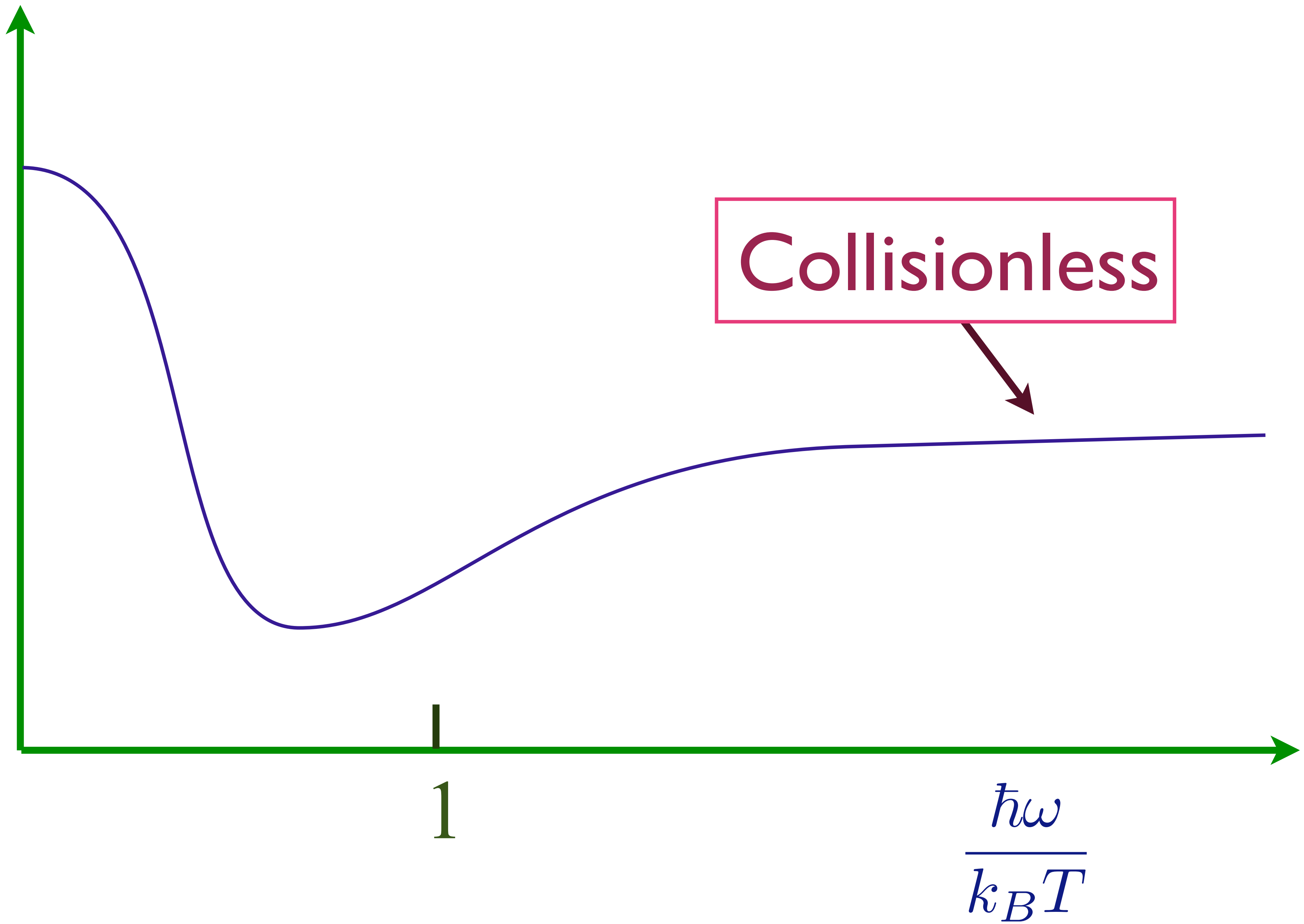

K. Damle and S. Sachdev, 1997

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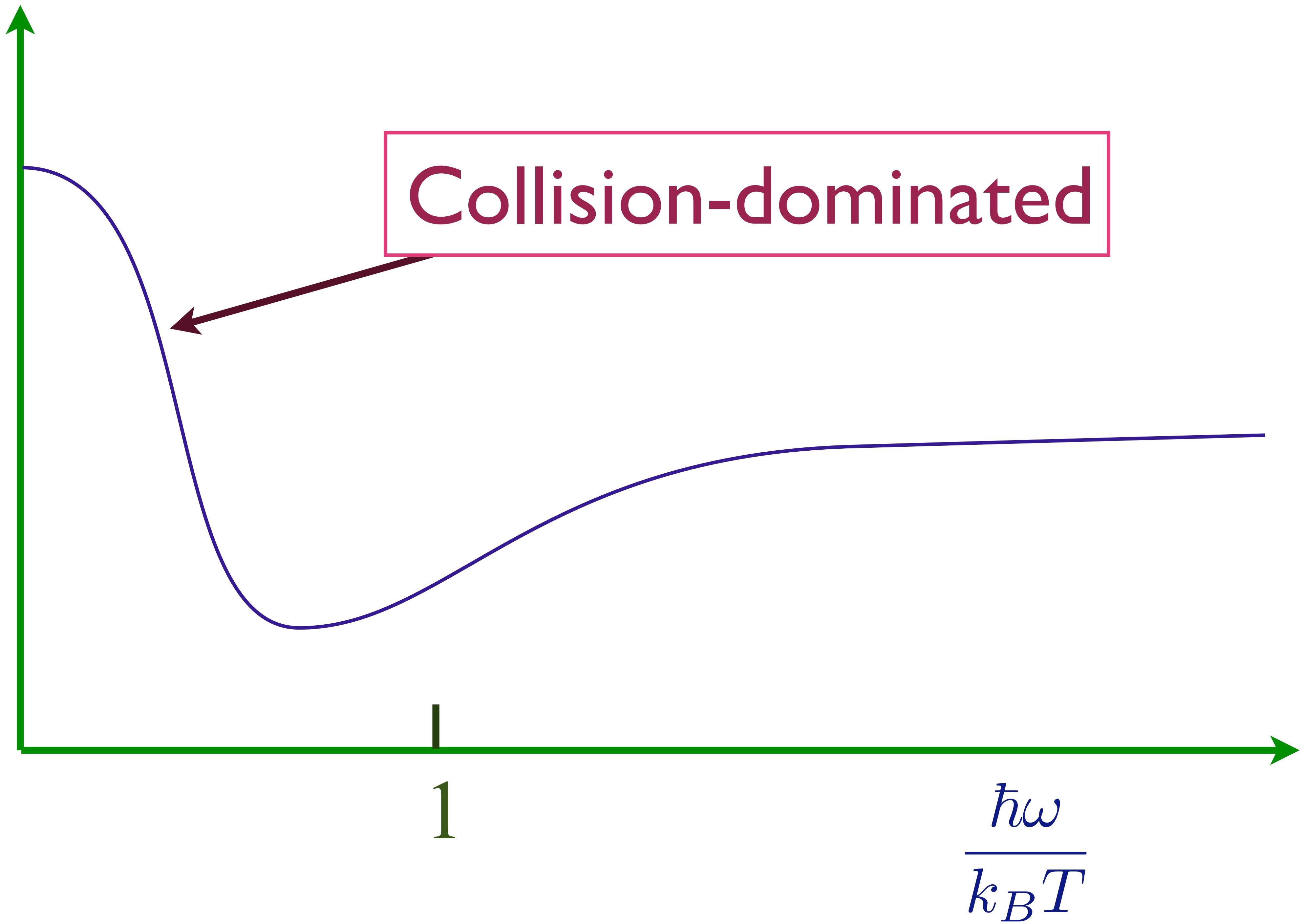

K. Damle and S. Sachdev, 1997

Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

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$$\mathcal{S} = \int d^3x \Big[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \Big]$$

<u>is dual to</u>

Using the vortex quasiparticle excitations of the superfluid $\sim \varphi$

$$\mathcal{S}_{\mathrm{dual}} = \int d^3x \Big[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

- Now add Dirac fermions, generalize the gauge group to $\mathrm{SU}(N)$, and allow maximal supersymmetry in 2+1 dimensions.

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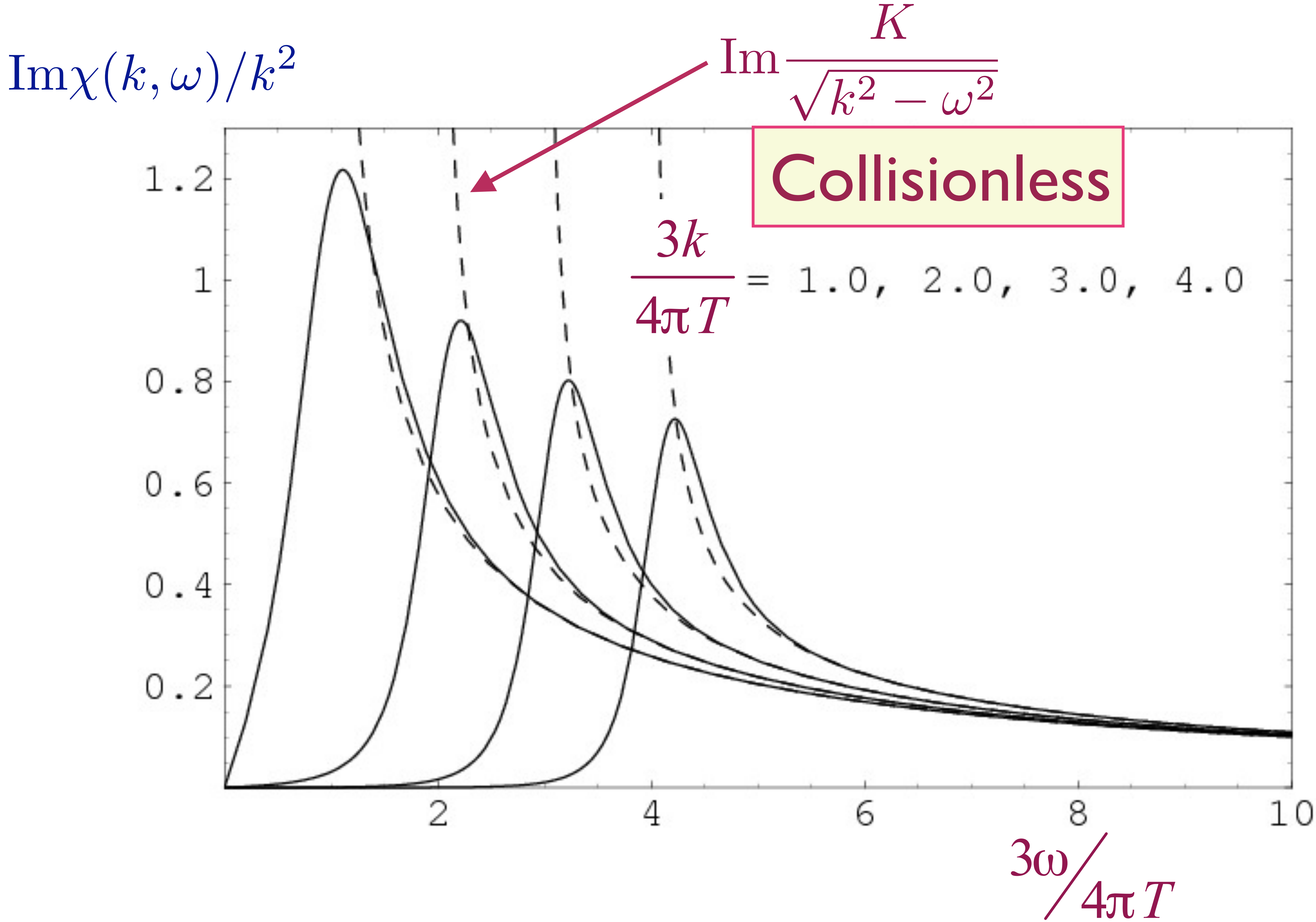
- Critical conductivity $\Sigma = \sqrt{2}N^{3/2}/3$ ("self-dual" value).

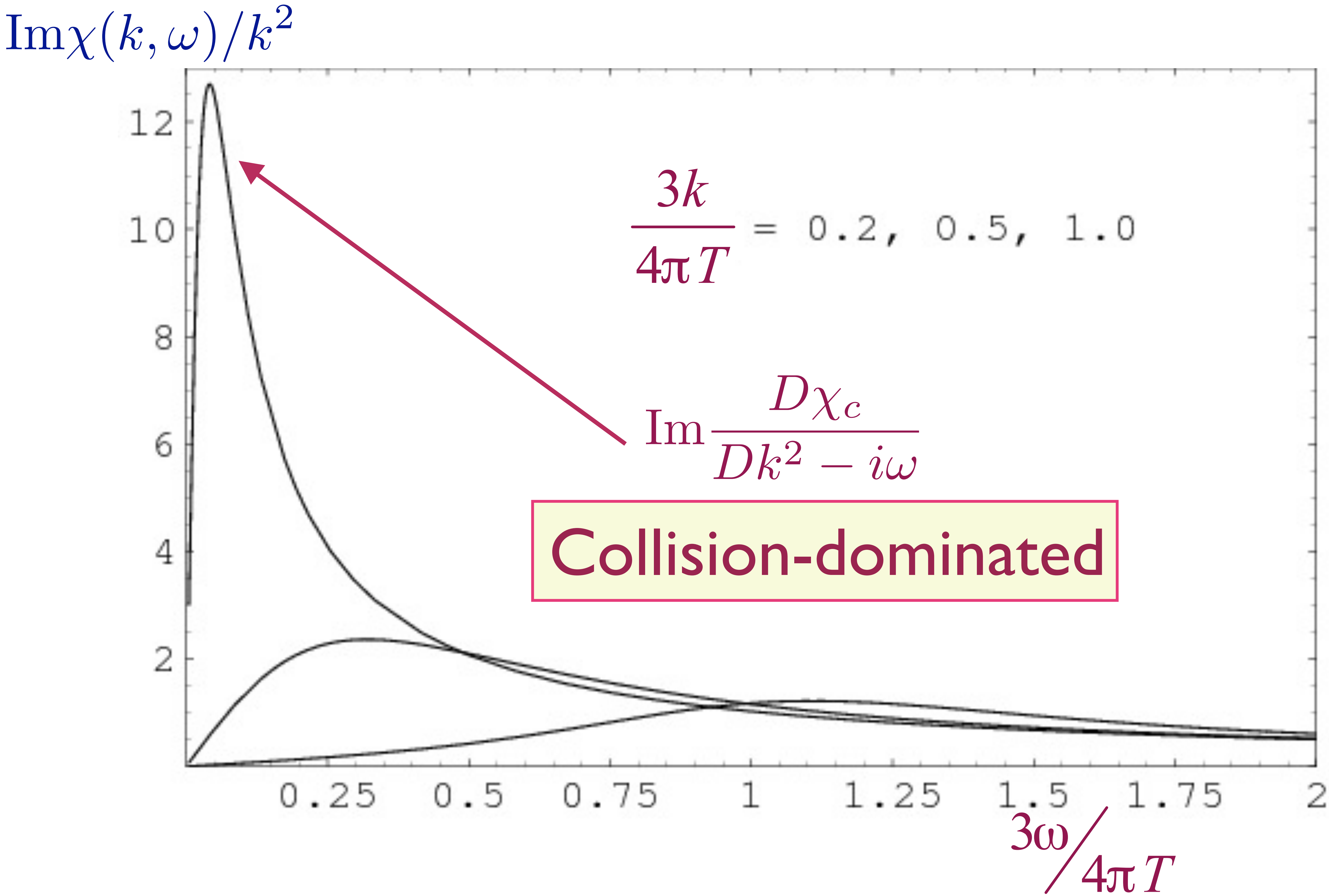
- For boson-vortex system, self-dual value is $\Sigma = 1$, closed to the observed values. Self-dual values are obtained for all models with simple gravity duals, analogous to $\eta/s = \hbar/(4\pi k_B)$.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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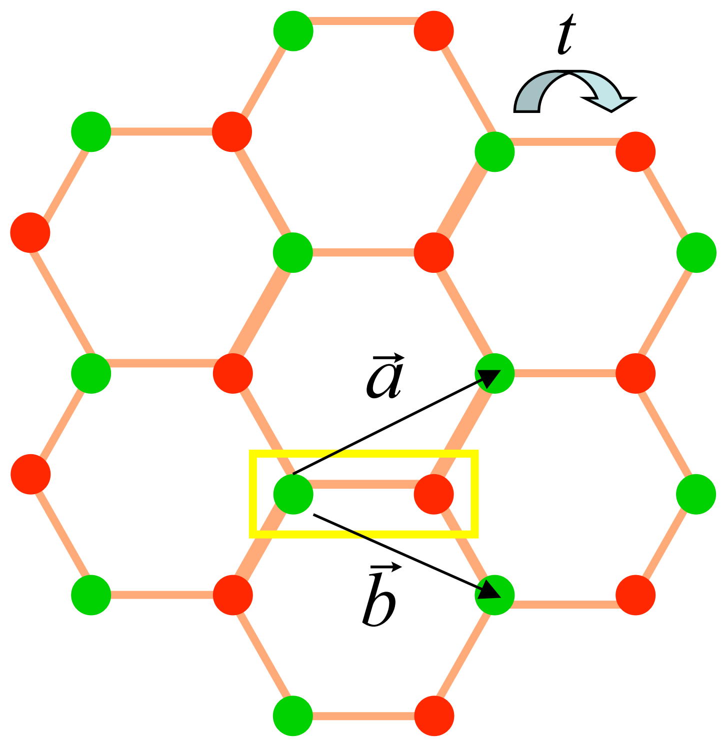
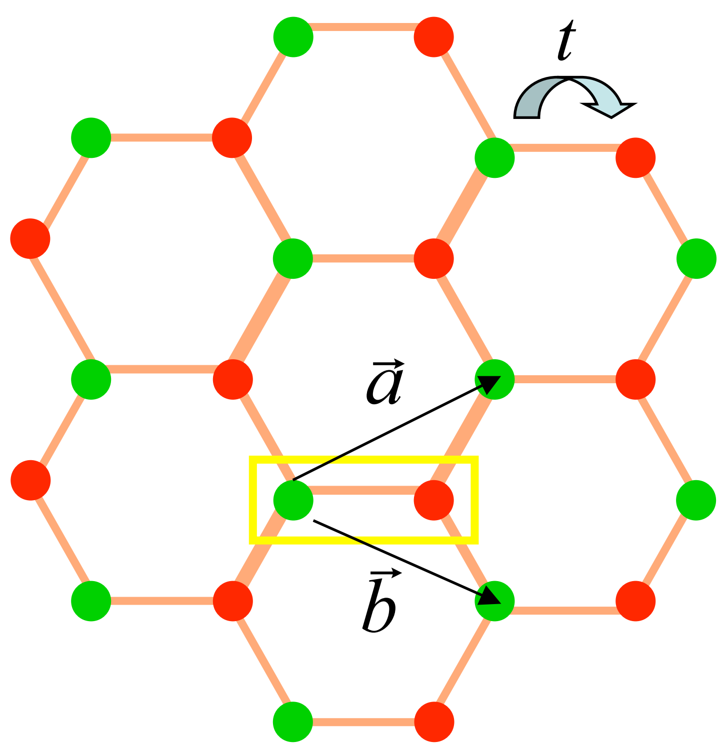
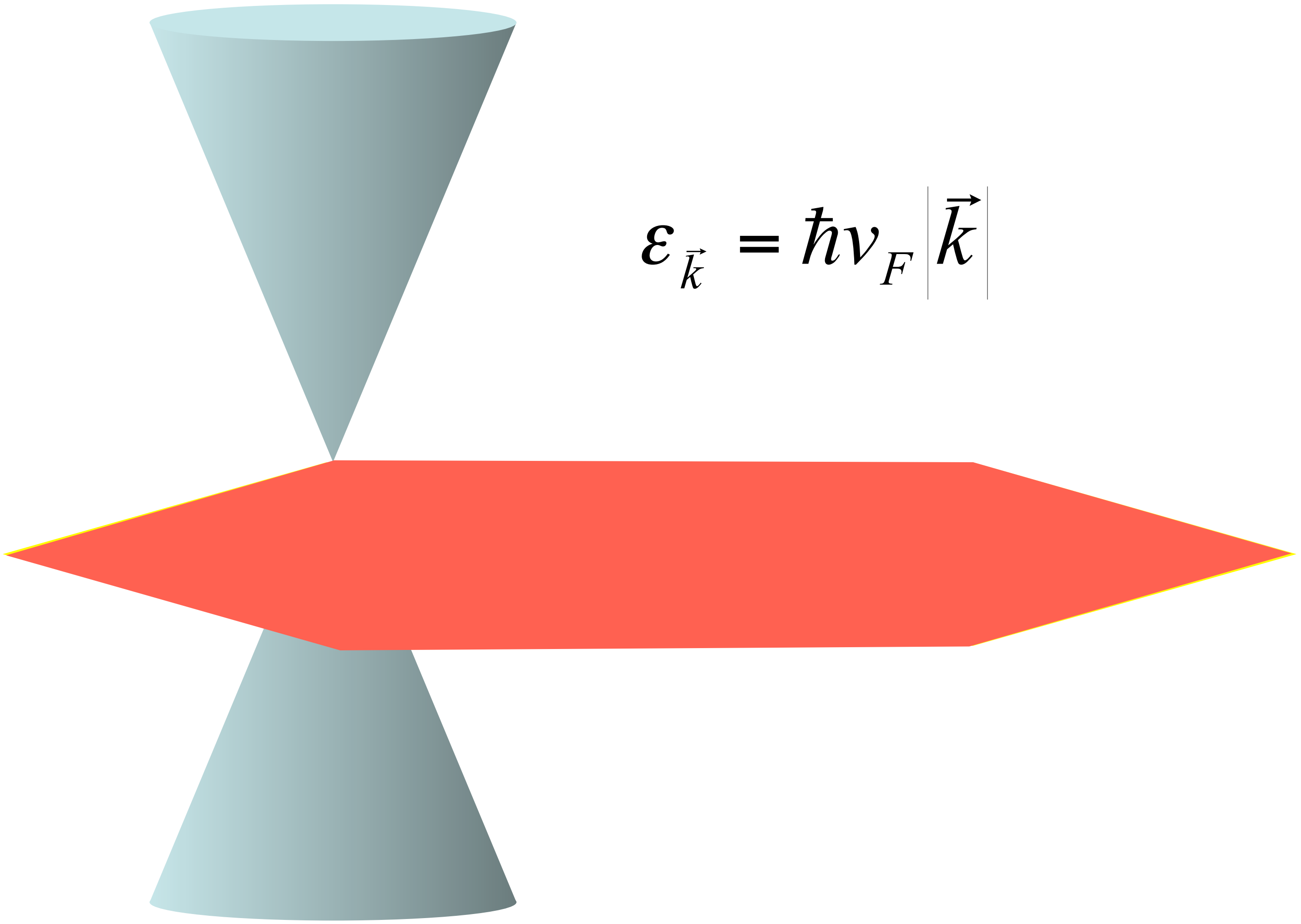
Graphene

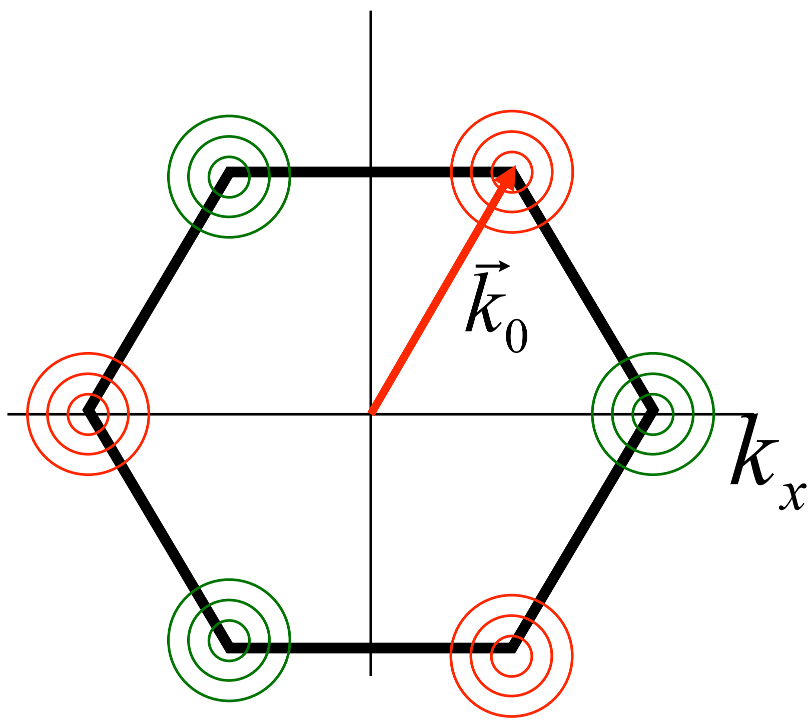


Graphene



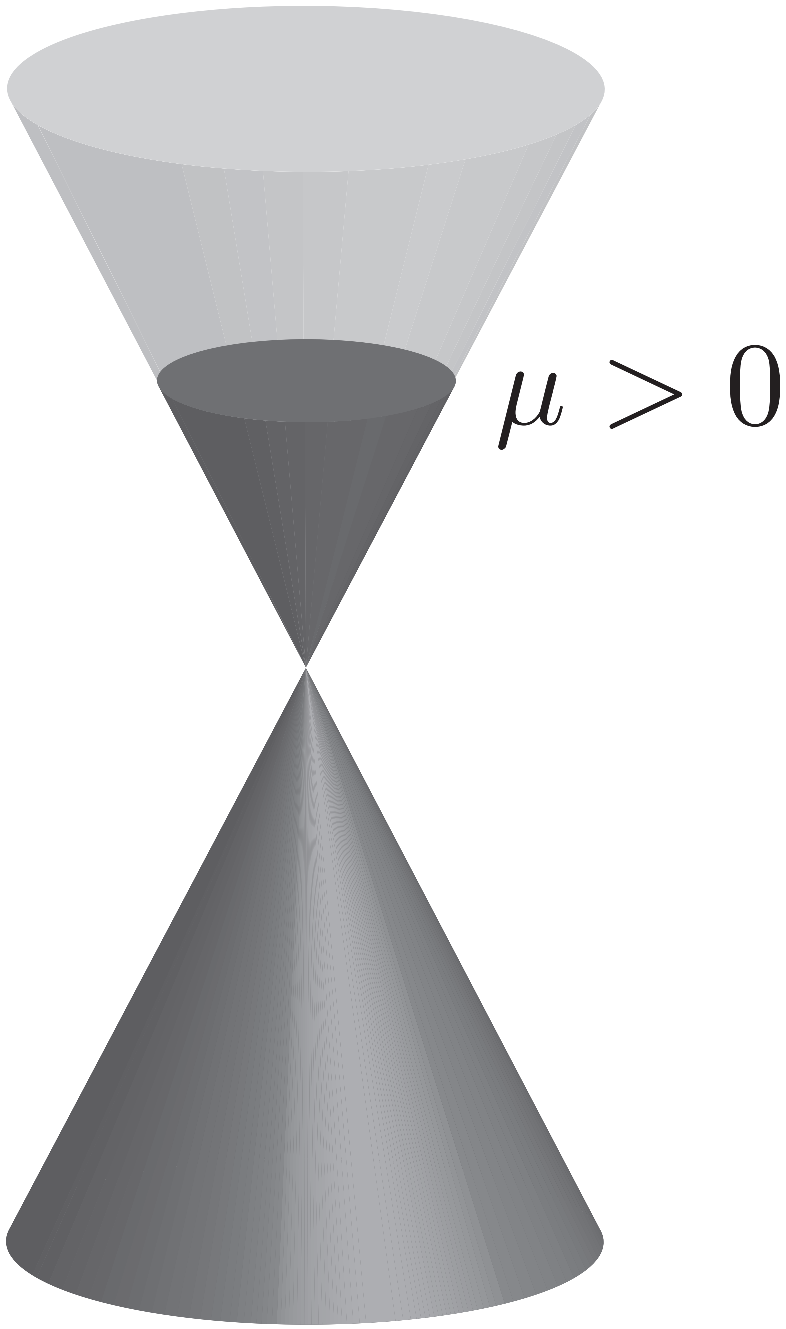


$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$



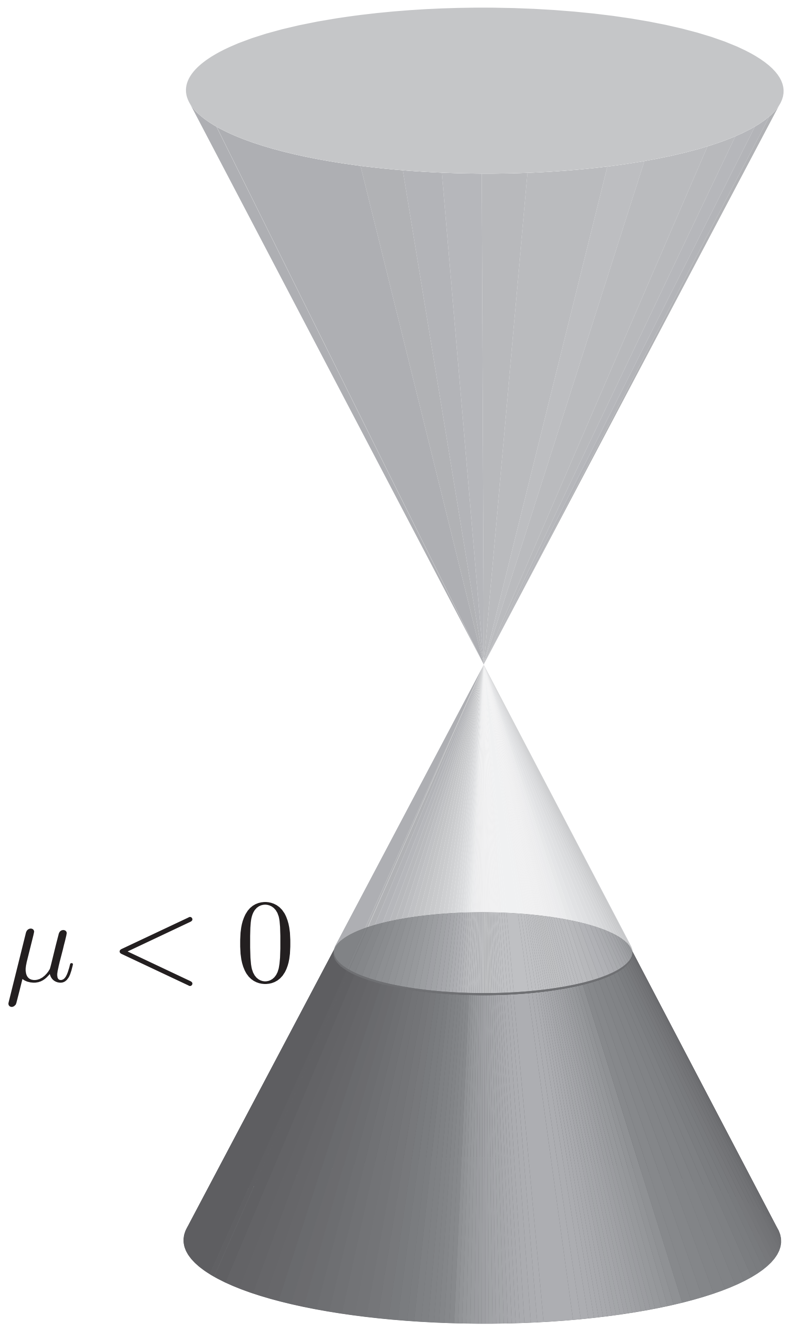
Conical Dirac dispersion

Quantum phase transition in graphene tuned by a gate voltage

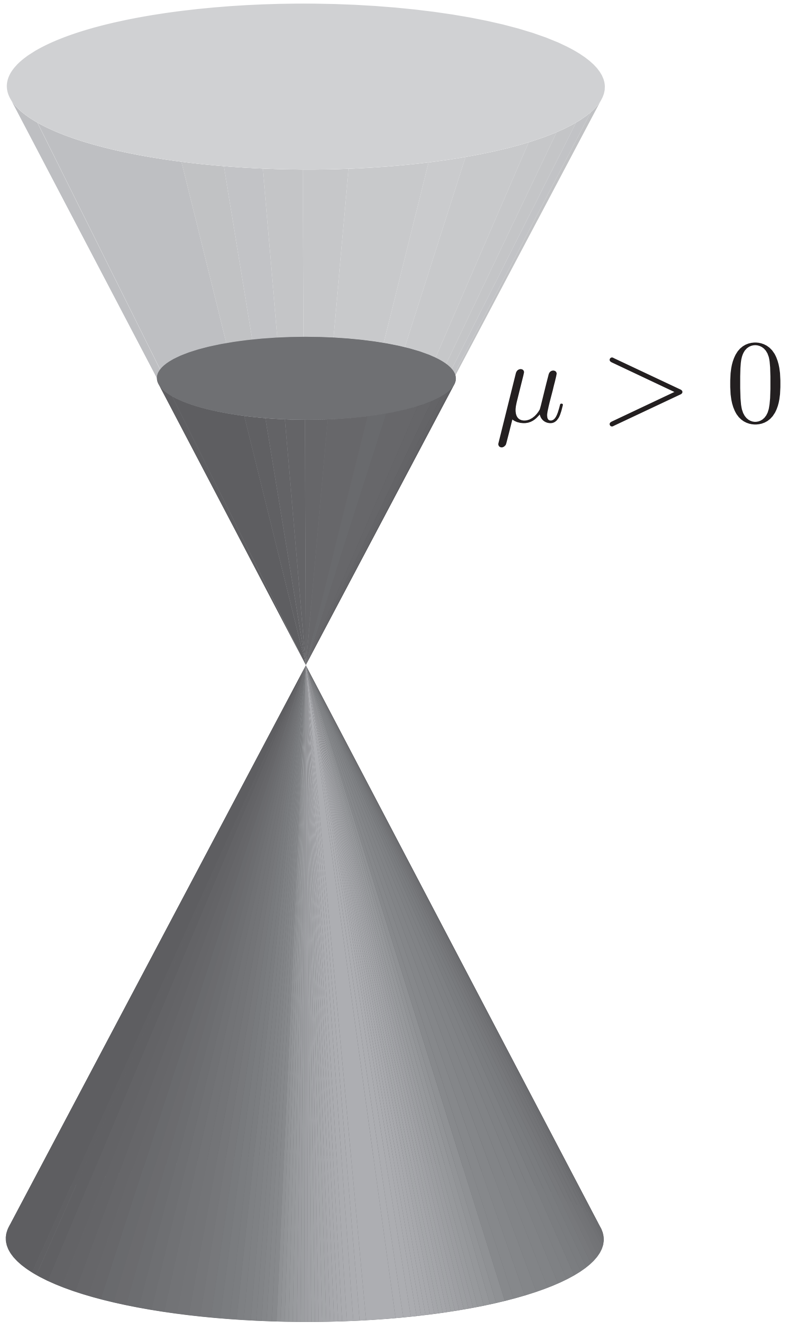


Electron
Fermi surface

Quantum phase transition in graphene tuned by a gate voltage



Hole
Fermi surface



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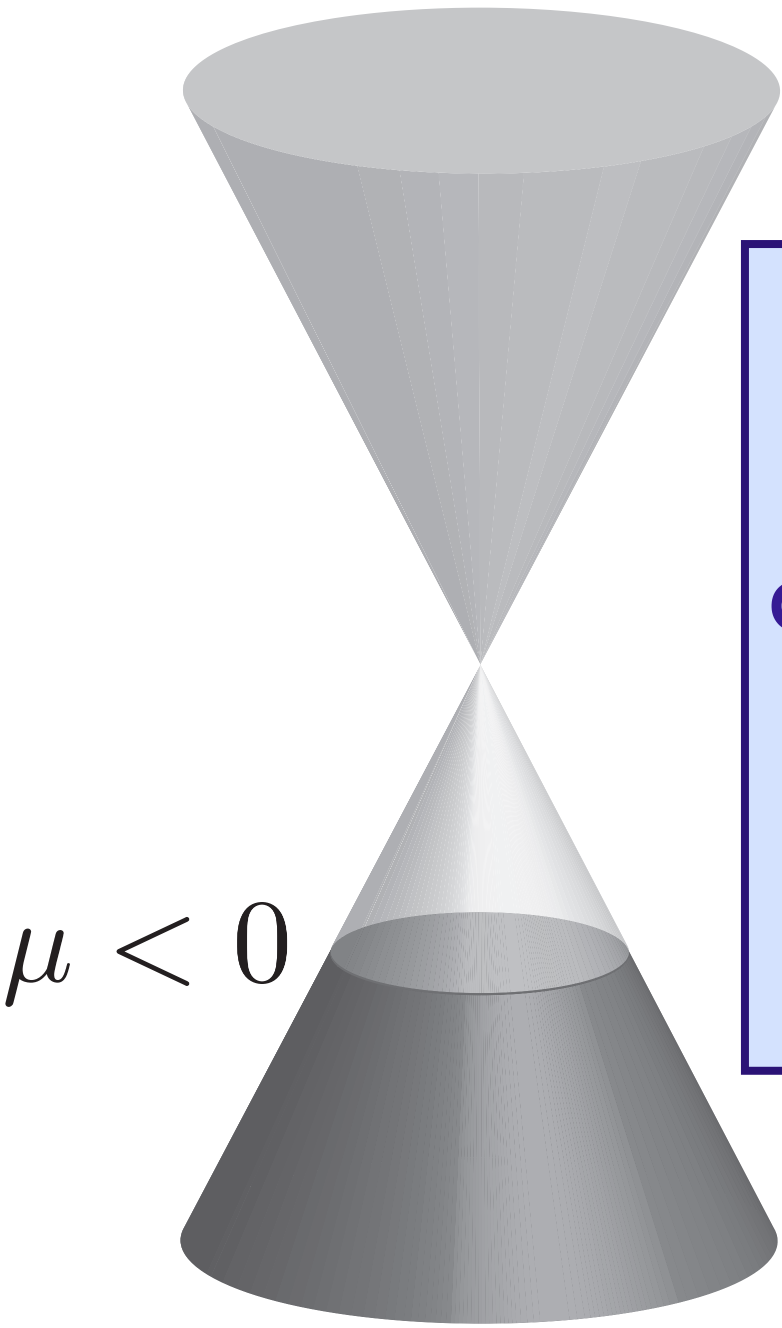
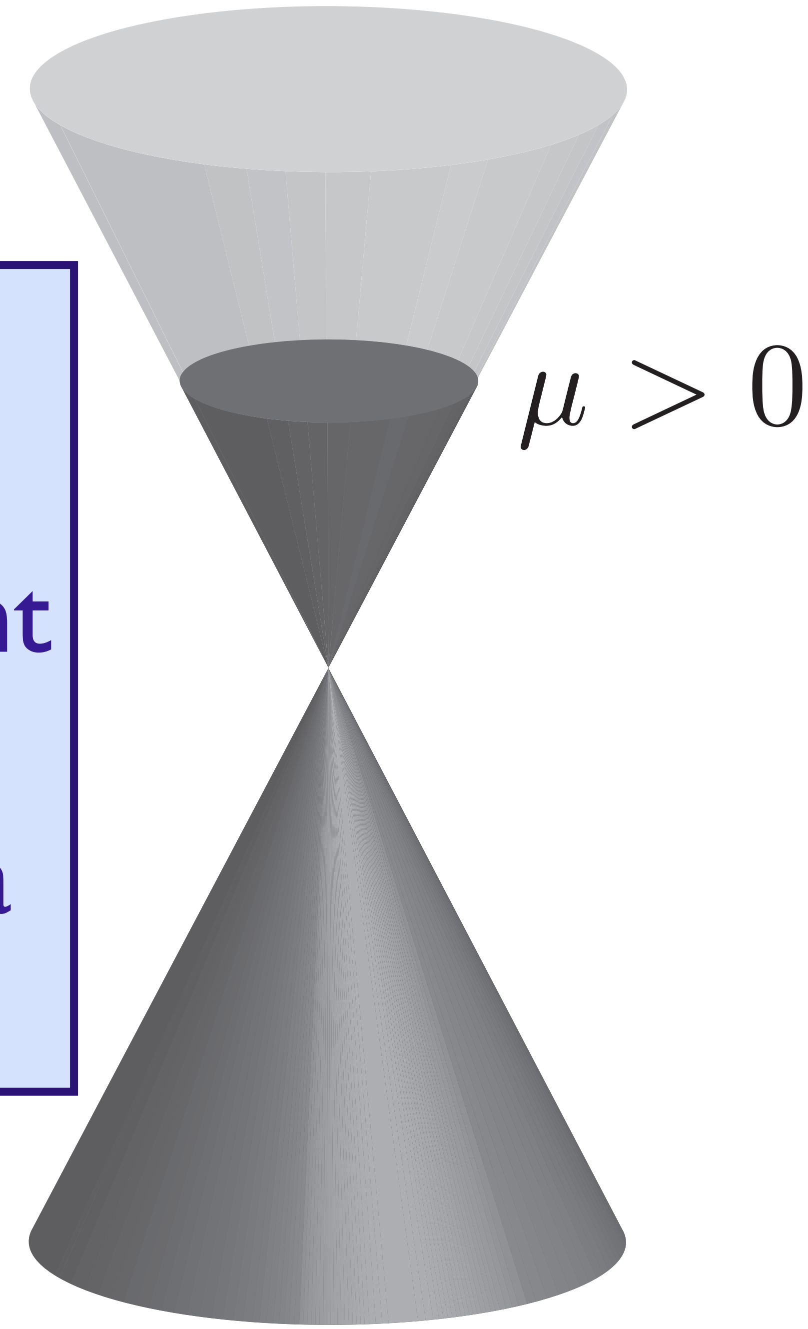


There must be an intermediate quantum critical point where the Fermi surfaces reduce to a Dirac point



Hole
Fermi surface

Electron
Fermi surface

Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, ψ_σ, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r') \end{aligned}$$

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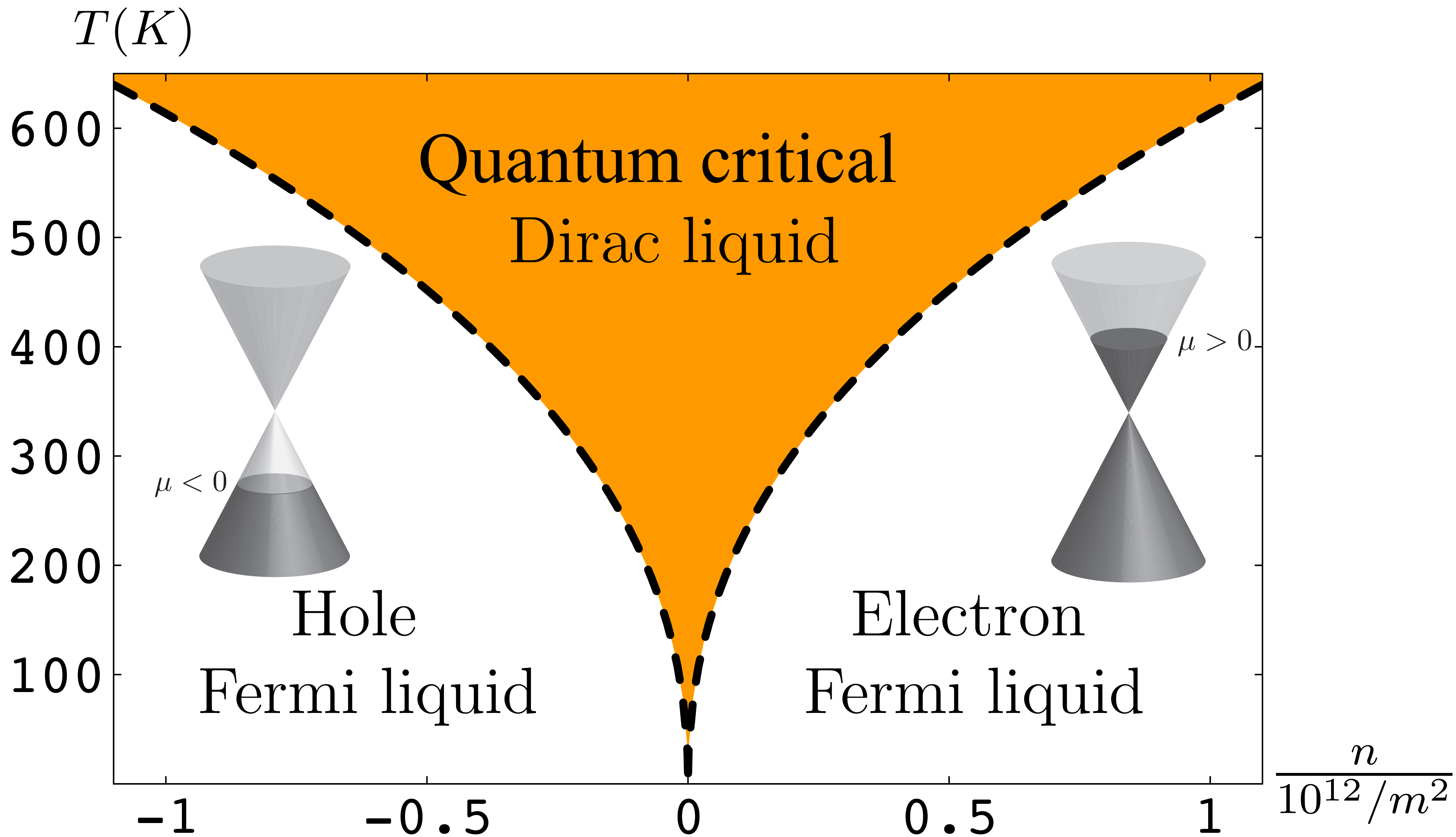
Dimensionless "fine-structure" constant $\alpha = e^2/(\hbar v_F)$.
RG flow of α:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$

Quantum phase transition in graphene



Quantum critical transport in graphene

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

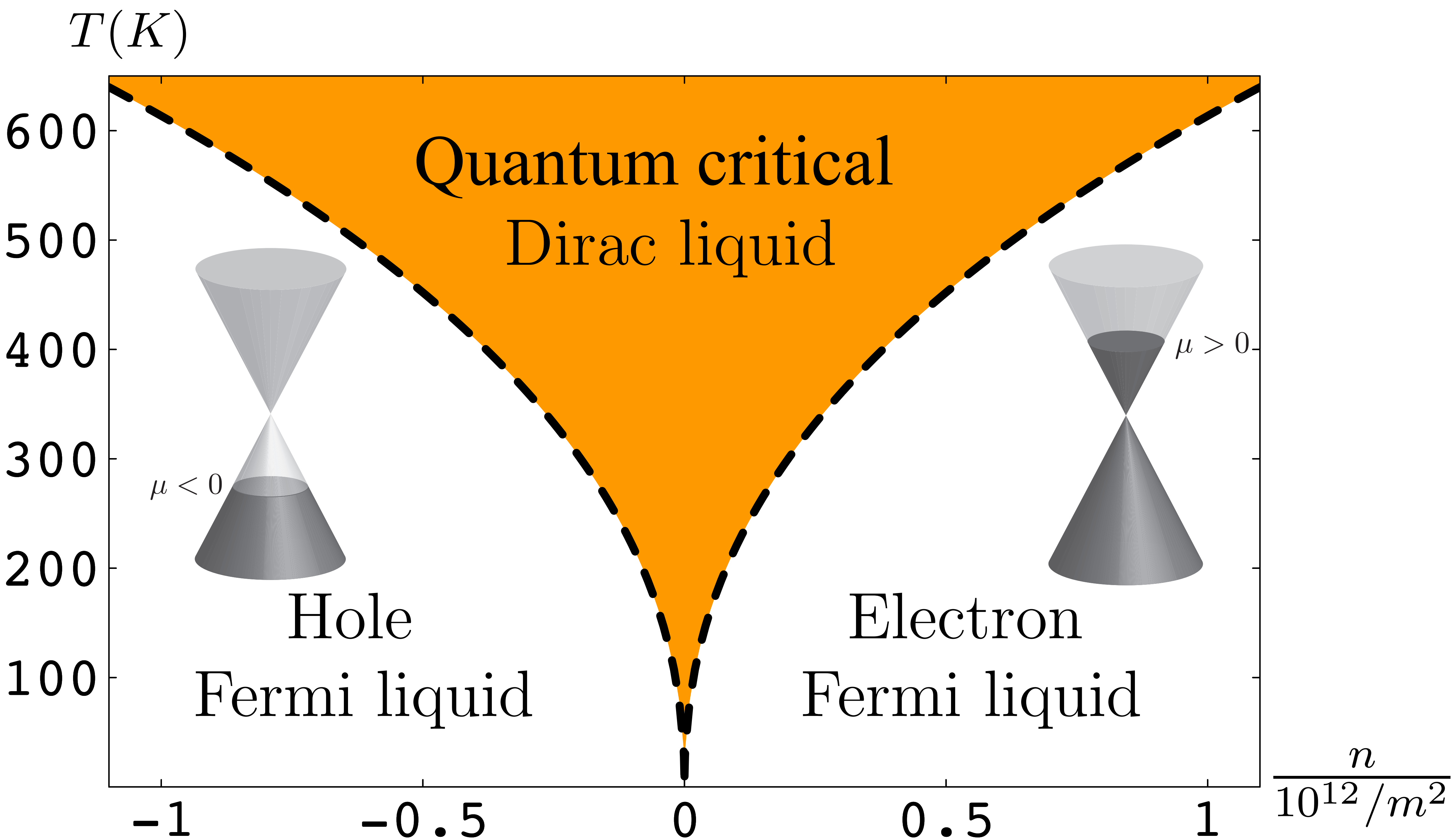
$$\frac{\eta}{s} = \frac{\hbar}{k_B \alpha^2(T)} \times 0.130$$

where the "fine structure constant" is

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

L. Fritz, J. Schmalian, M. Müller and S. Sachdev, *Physical Review B* **78**, 085416 (2008)
M. Müller, J. Schmalian, and L. Fritz, *Physical Review Letters* **103**, 025301 (2009)

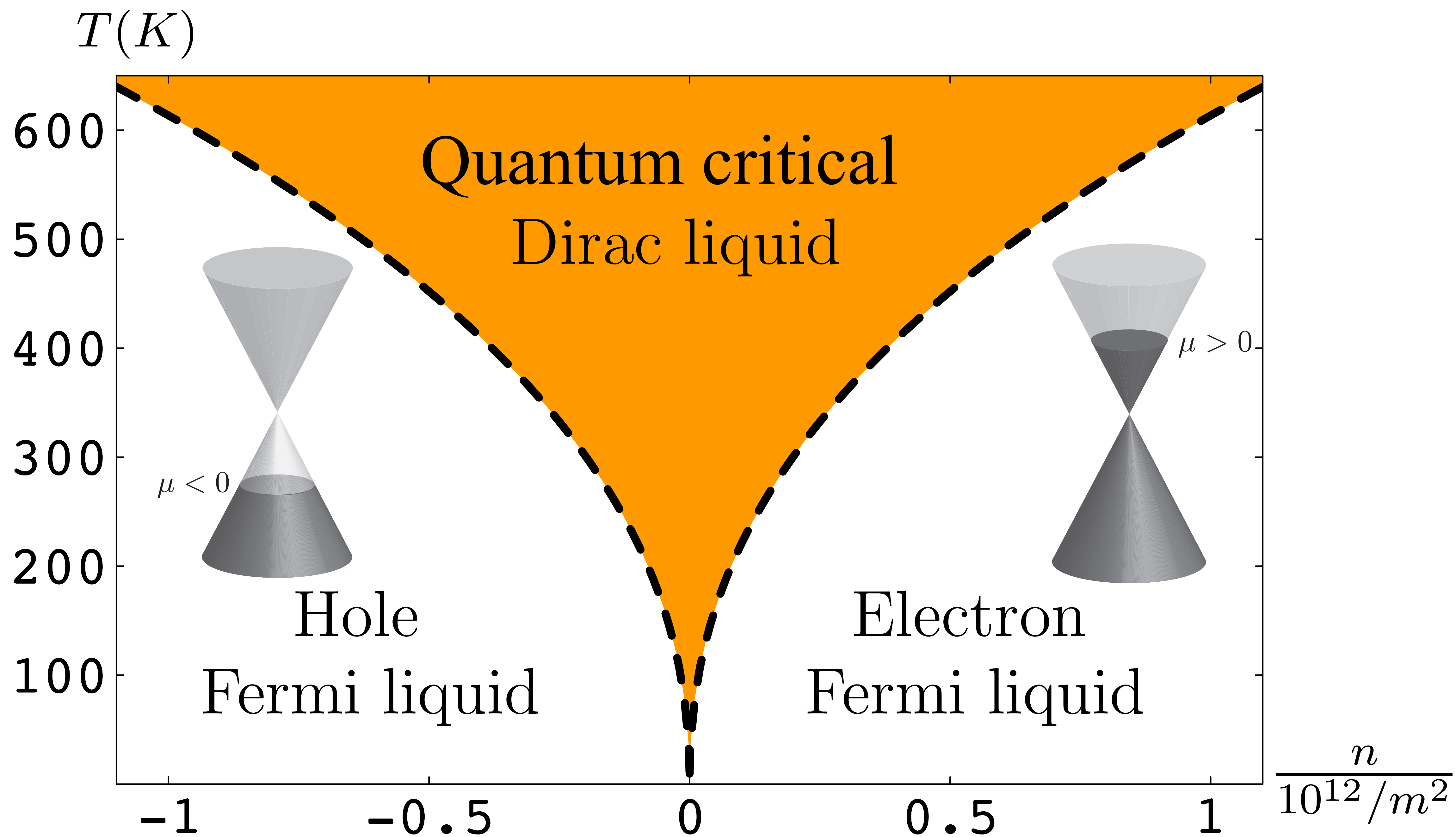
Previously unsolved: general quantum critical transport theory for arbitrary μ, applied magnetic field B, and small impurity density, and general ω/T.



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Previously unsolved: general quantum critical transport theory for arbitrary μ, applied magnetic field B, and small impurity density, and general ω/T.

$\Rightarrow$ maps onto quasinormal modes of a Reissner-Nordstorm black hole in AdS_4.



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The **same** results were later obtained from the equations of generalized relativistic magnetohydrodynamics, *and* from a solution of the quantum Boltzmann equation.

So the results apply to experiments on graphene, the cuprates, *and* to the dynamics of black holes.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

As a simple example, in zero magnetic field, we can write the electrical conductivity as

$$\sigma = \sigma_Q + \frac{e^{*2}\rho^2 v^2}{\varepsilon + P}\pi\delta(\omega)$$

where σ_Q is the universal conductivity of the CFT, ρ is the charge density, ε is the energy density and P is the pressure.

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The same quantities also determine a "Wiedemann-Franz"-like relation for thermal conductivity, κ at $B = 0$

$$\kappa = \sigma_Q \left(\frac{k_B^2 T}{e^{*2}} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 .$$

At $B \neq 0$ and $\rho = 0$ we have a "Wiedemann-Franz" relation for "vortices"

$$\kappa = \frac{1}{\sigma_Q} k_B^2 T \left(\frac{v(\varepsilon + P)}{k_B T B} \right)^2 .$$

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

A second example: In an applied magnetic field B, the dynamic transport co-efficients exhibit a **hydrodynamic cyclotron resonance** at a frequency ω_c

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and damping constant γ

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}.$$

The same constants determine the **quasinormal frequency** of the Reissner-Nordstrom black hole.

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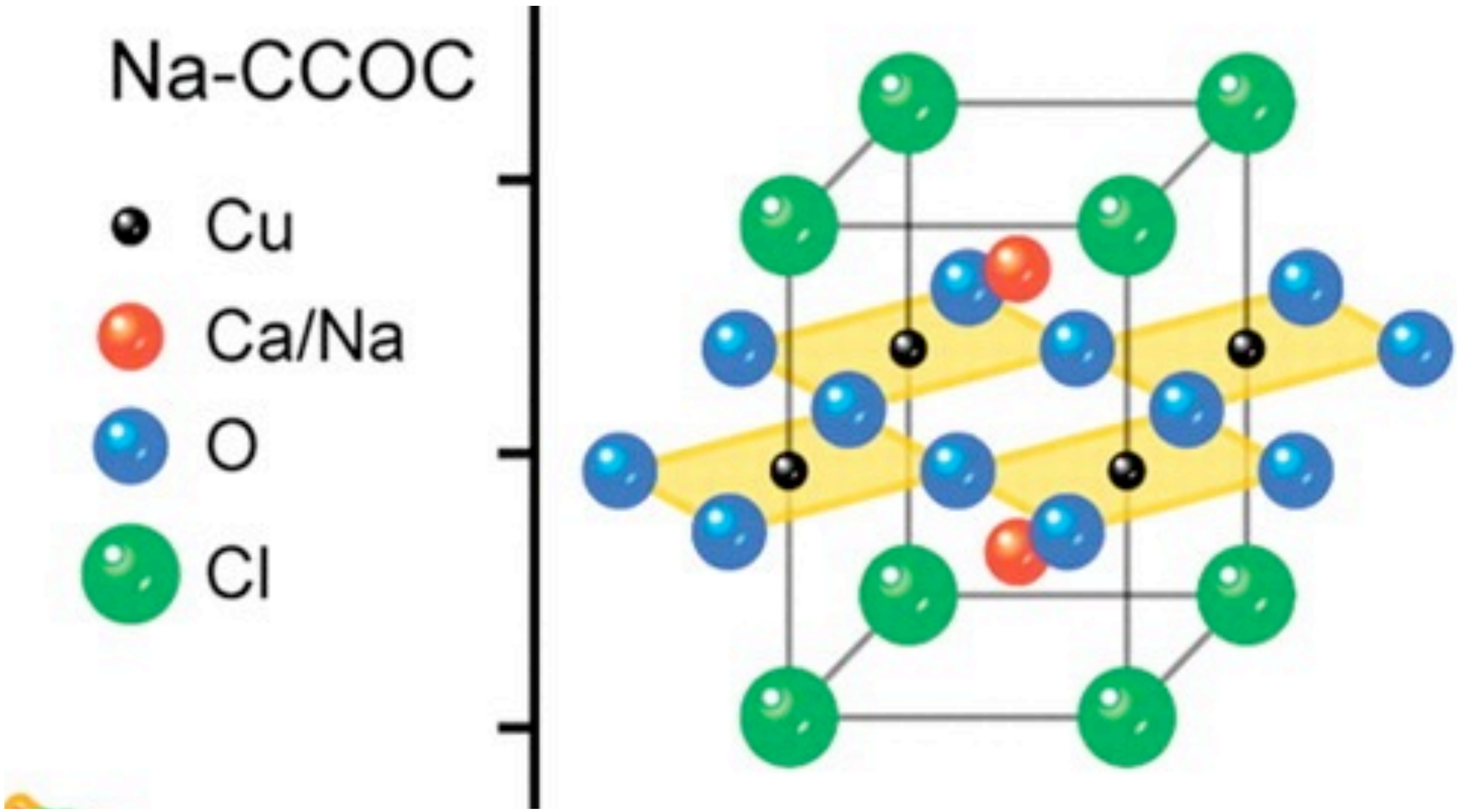
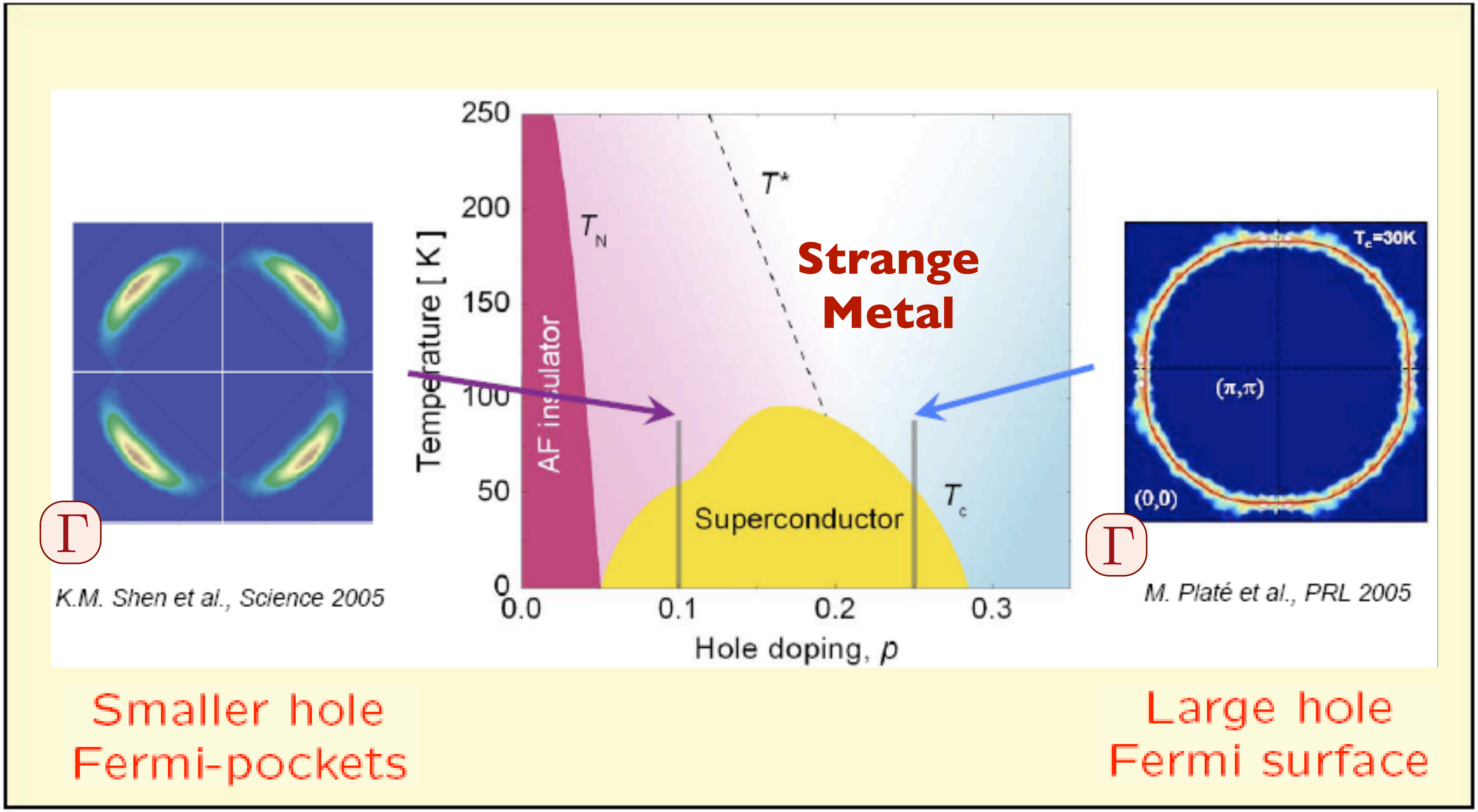
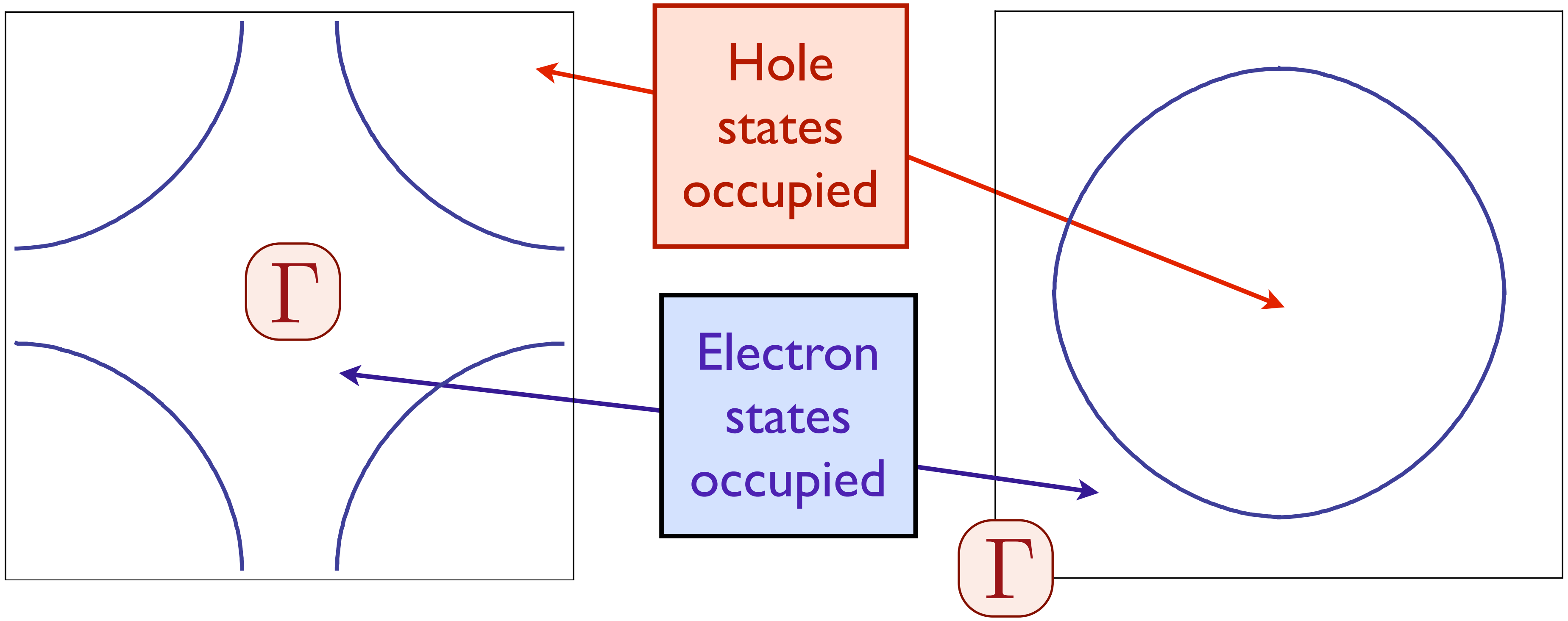
The cuprate superconductors



Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Fermi surface+antiferromagnetism



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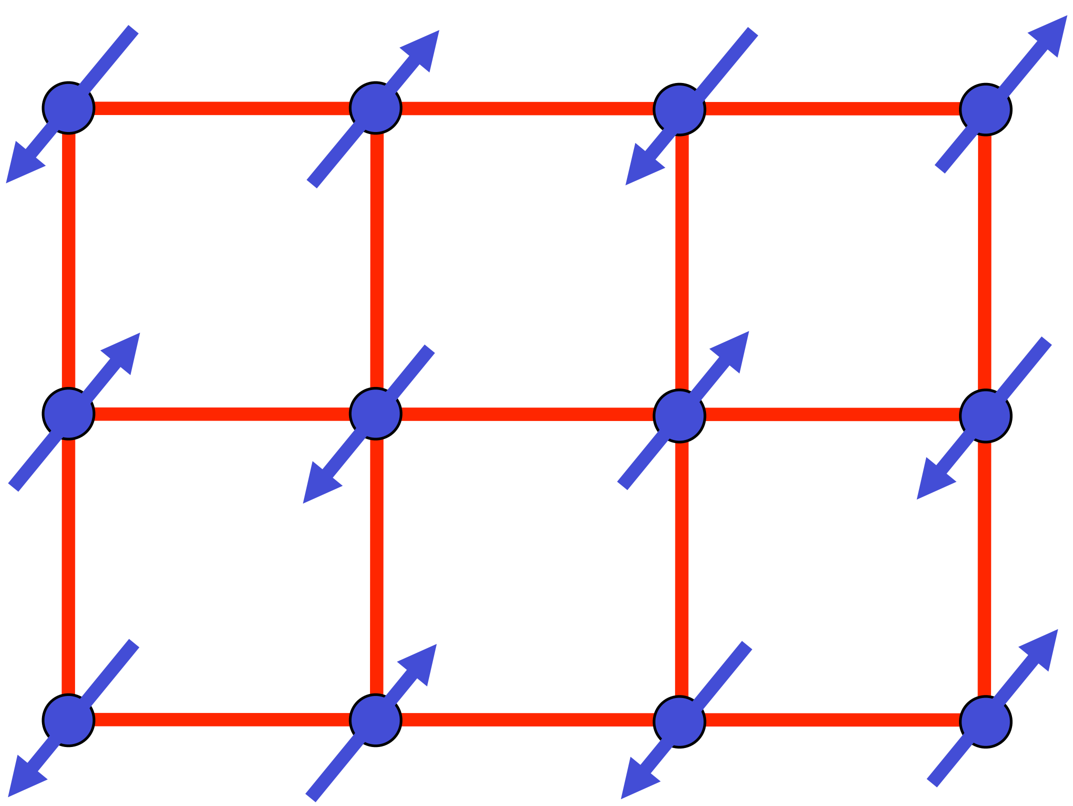


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

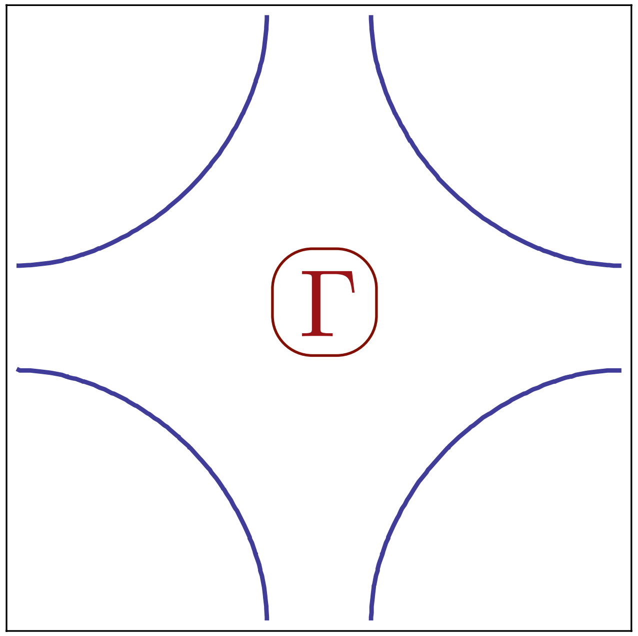
$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

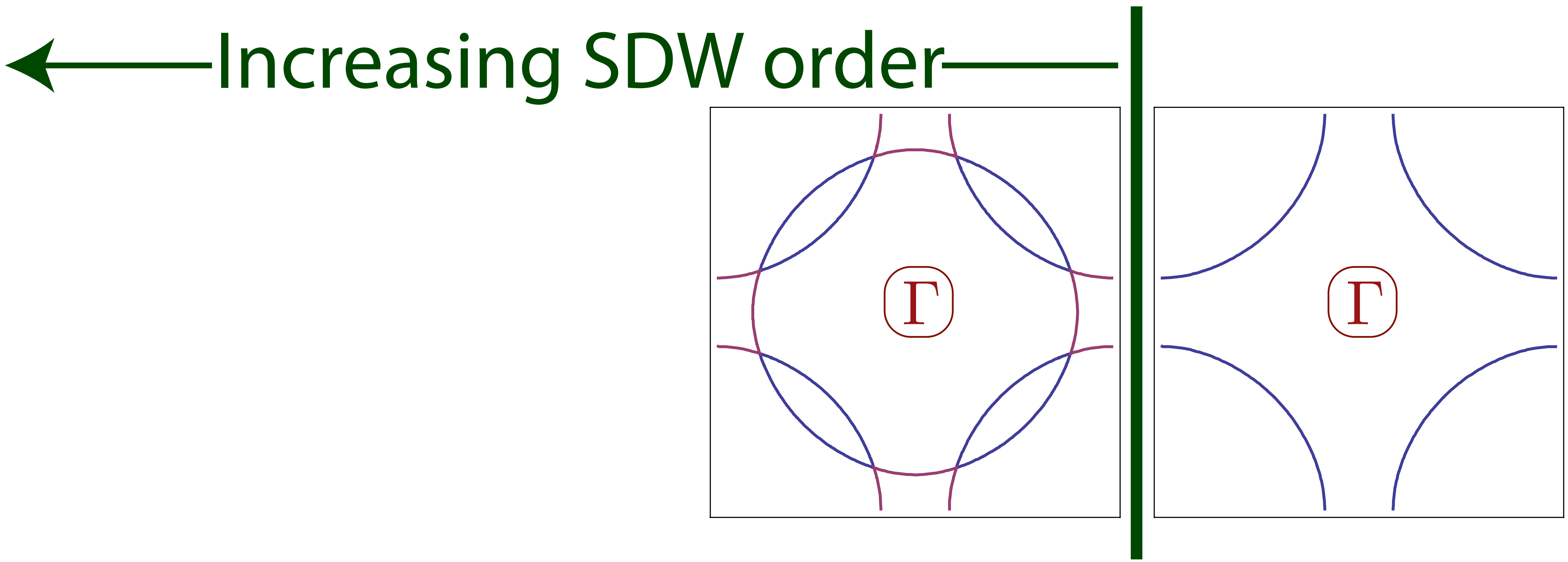
Hole-doped cuprates

Increasing SDW order



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



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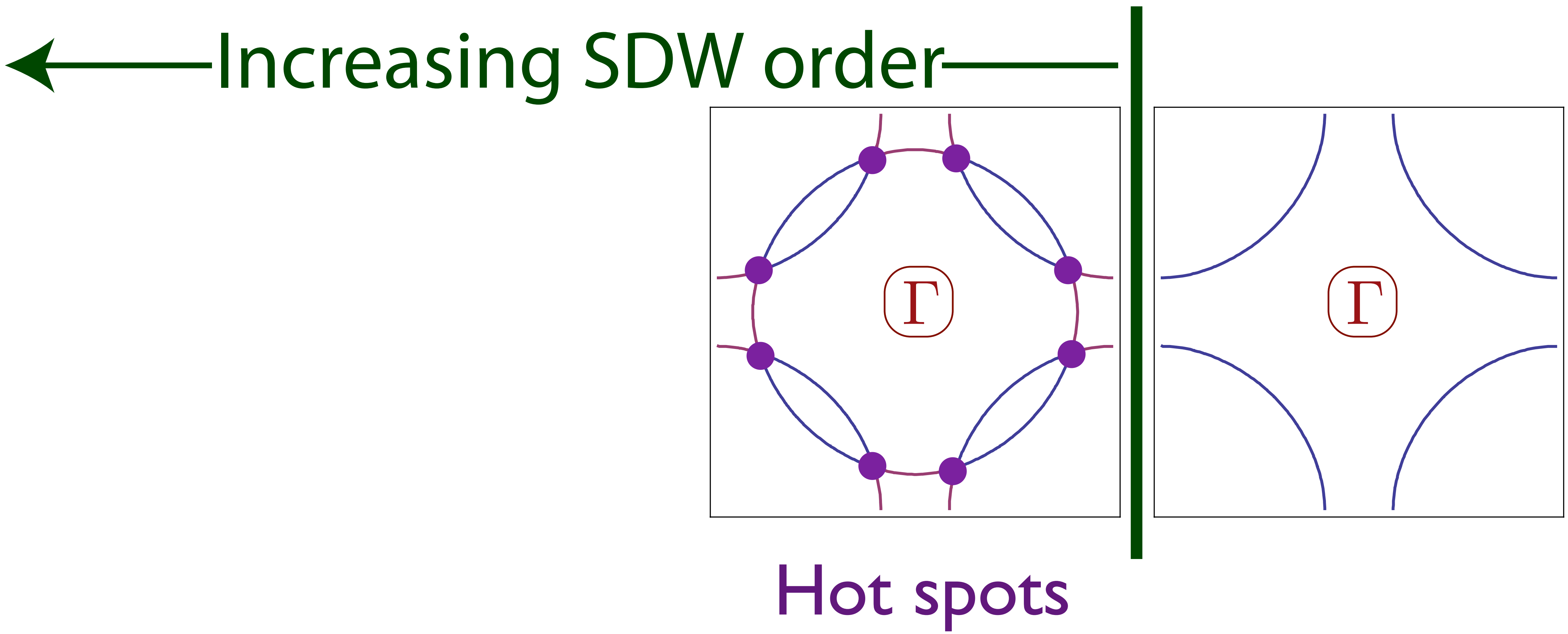

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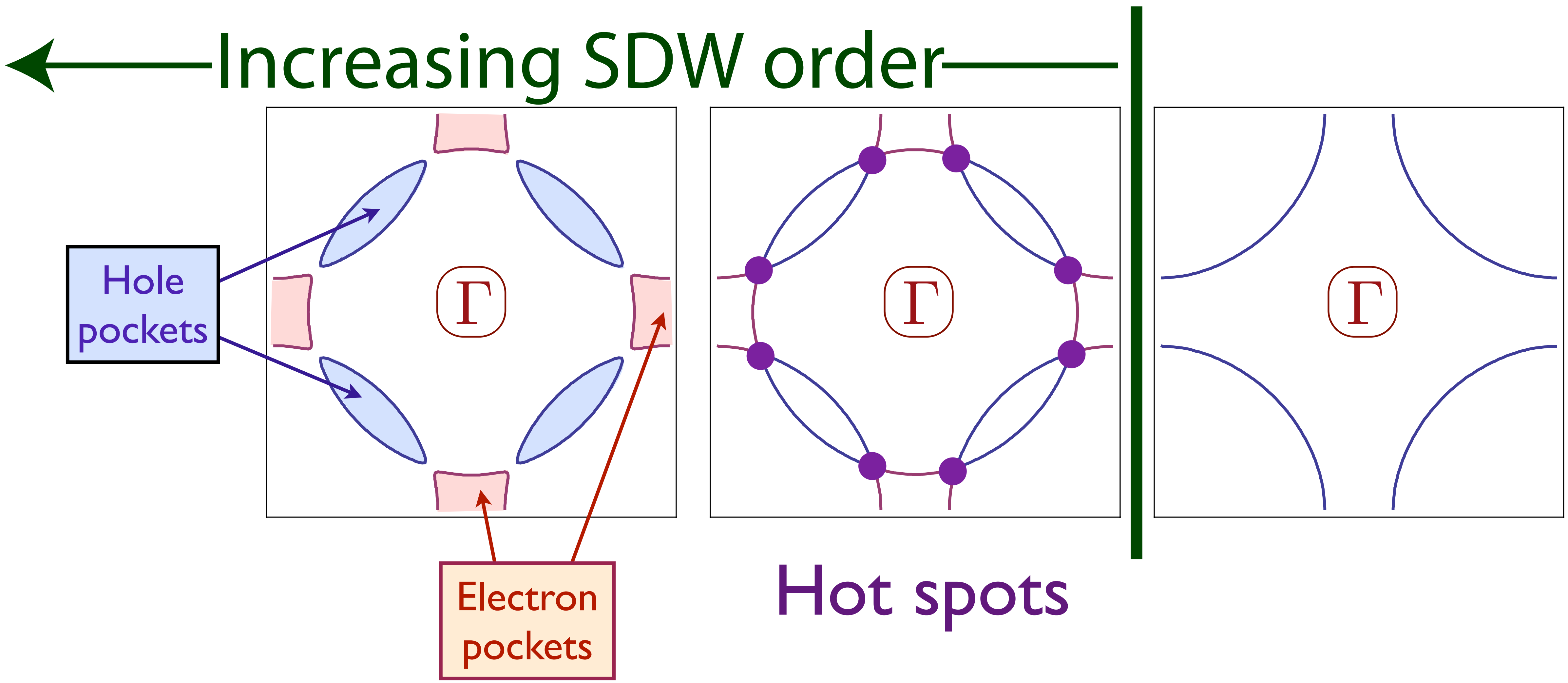

Fermi surface breaks up at hot spots into electron and hole "pockets"

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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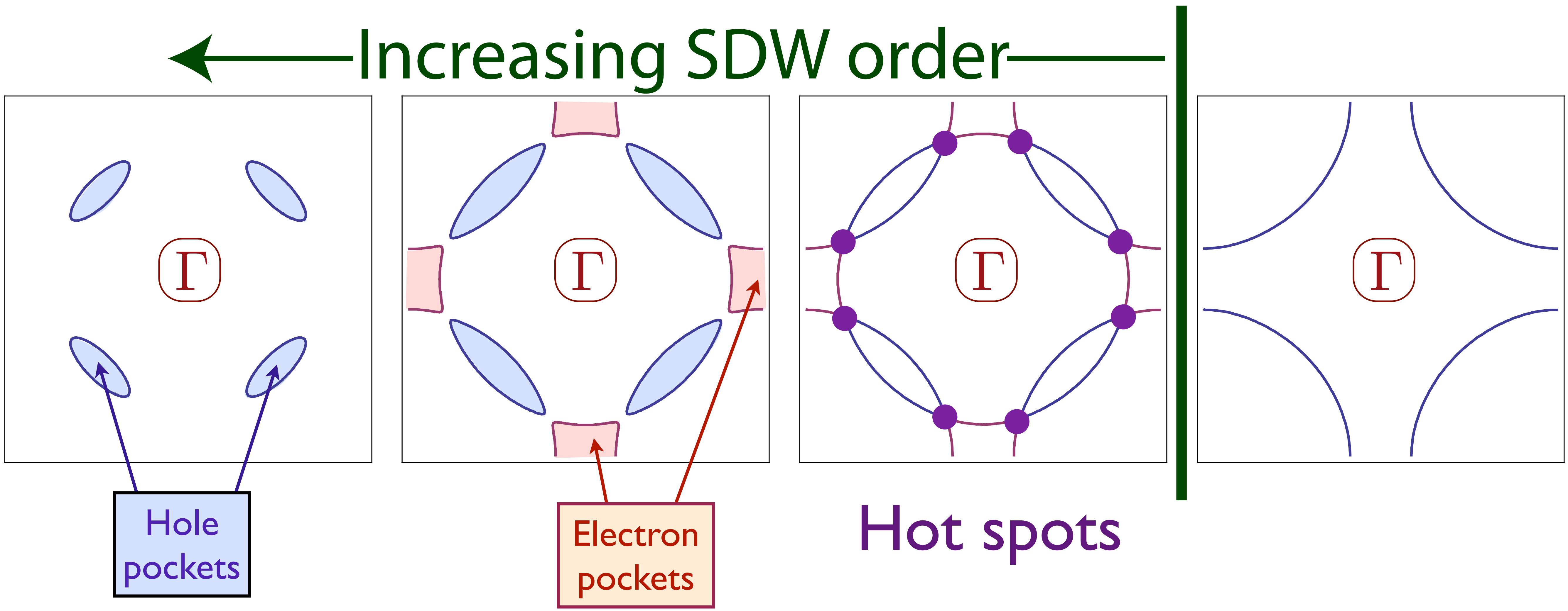
Hole-doped cuprates



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A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Evidence for small Fermi pockets

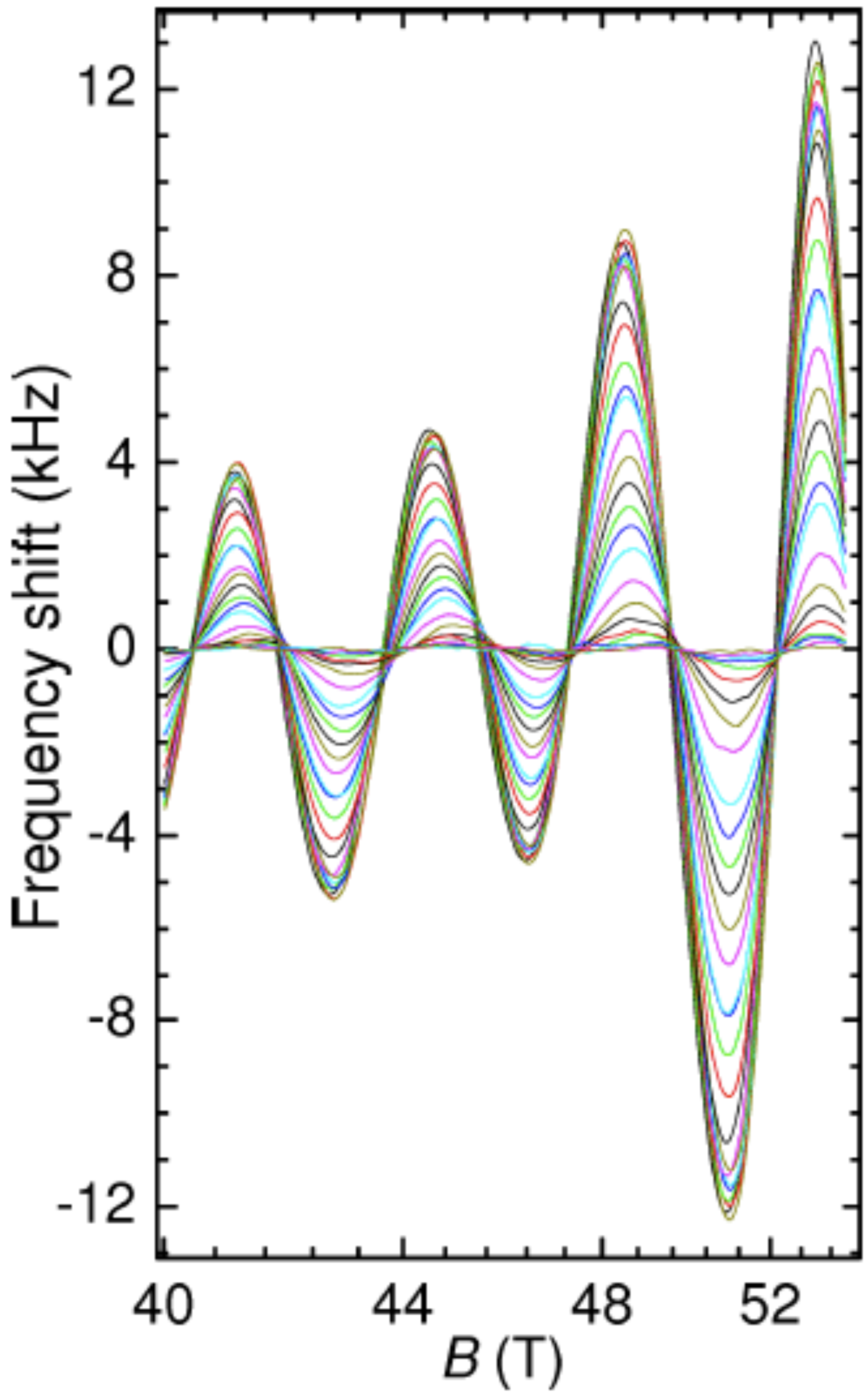


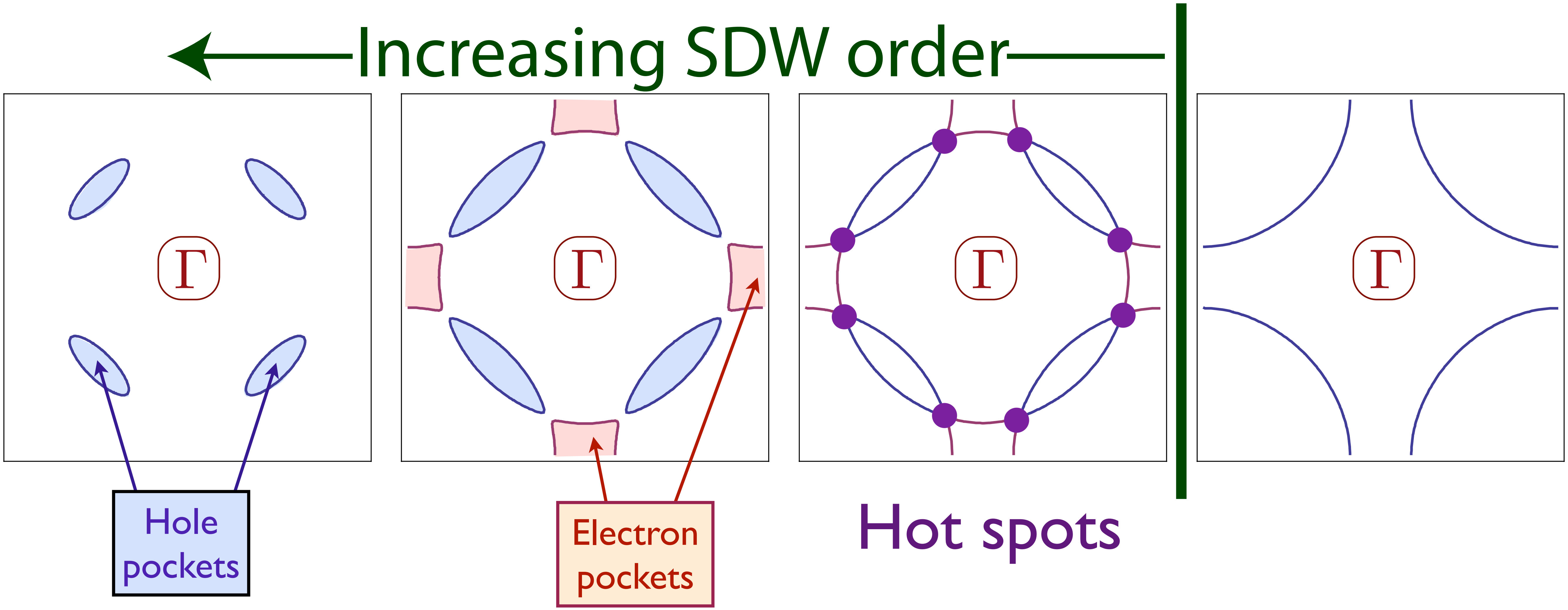
FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between T = 1 and 18 K. The actual T values are provided in Fig. 3.

Fermi liquid behaviour in an underdoped high Tc superconductor

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich

arXiv:0912.3022

Theory of underdoped cuprates



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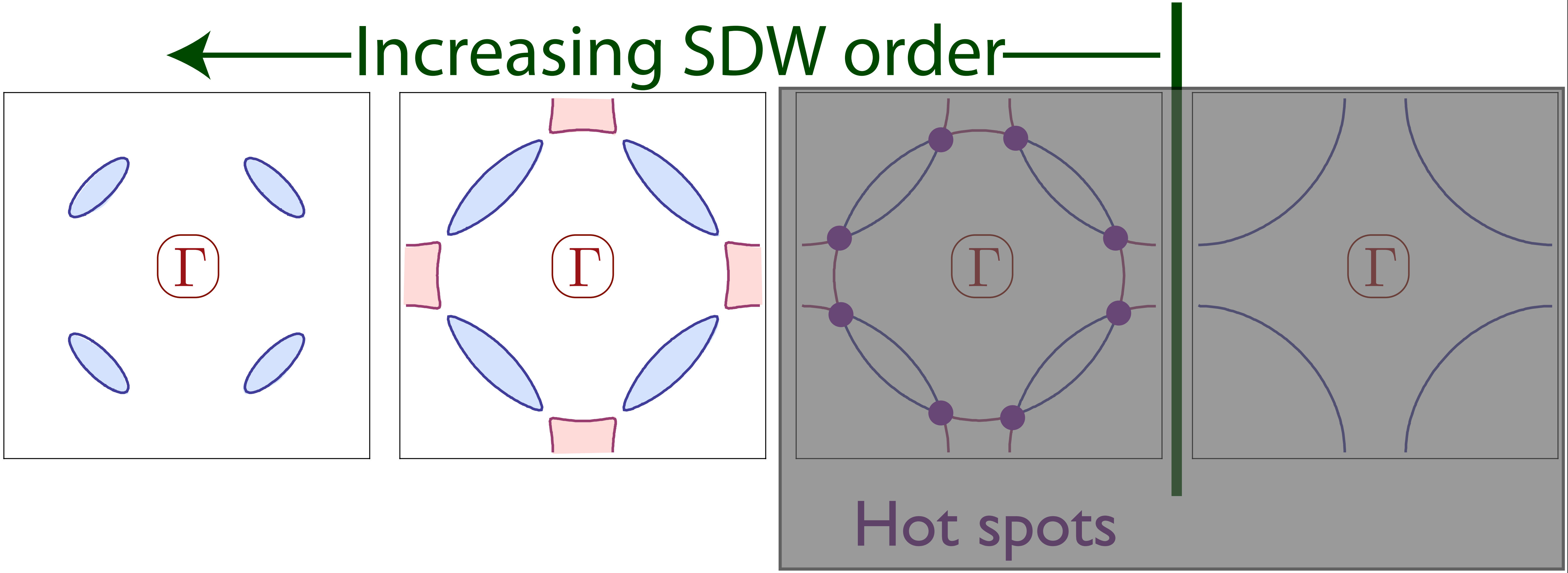
Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)
B. I. Shraiman and E. D. Siggia, *Physical Review Letters* **61**, 467 (1988).
J. R. Schrieffer, *Journal of Superconductivity* **17**, 539 (2004)

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \quad \text{or} \quad \hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

the theory is invariant under

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \ ; \ \psi_+ \rightarrow e^{-i\theta}\psi_+ \ ; \ \psi_- \rightarrow e^{i\theta}\psi_{.-}$$

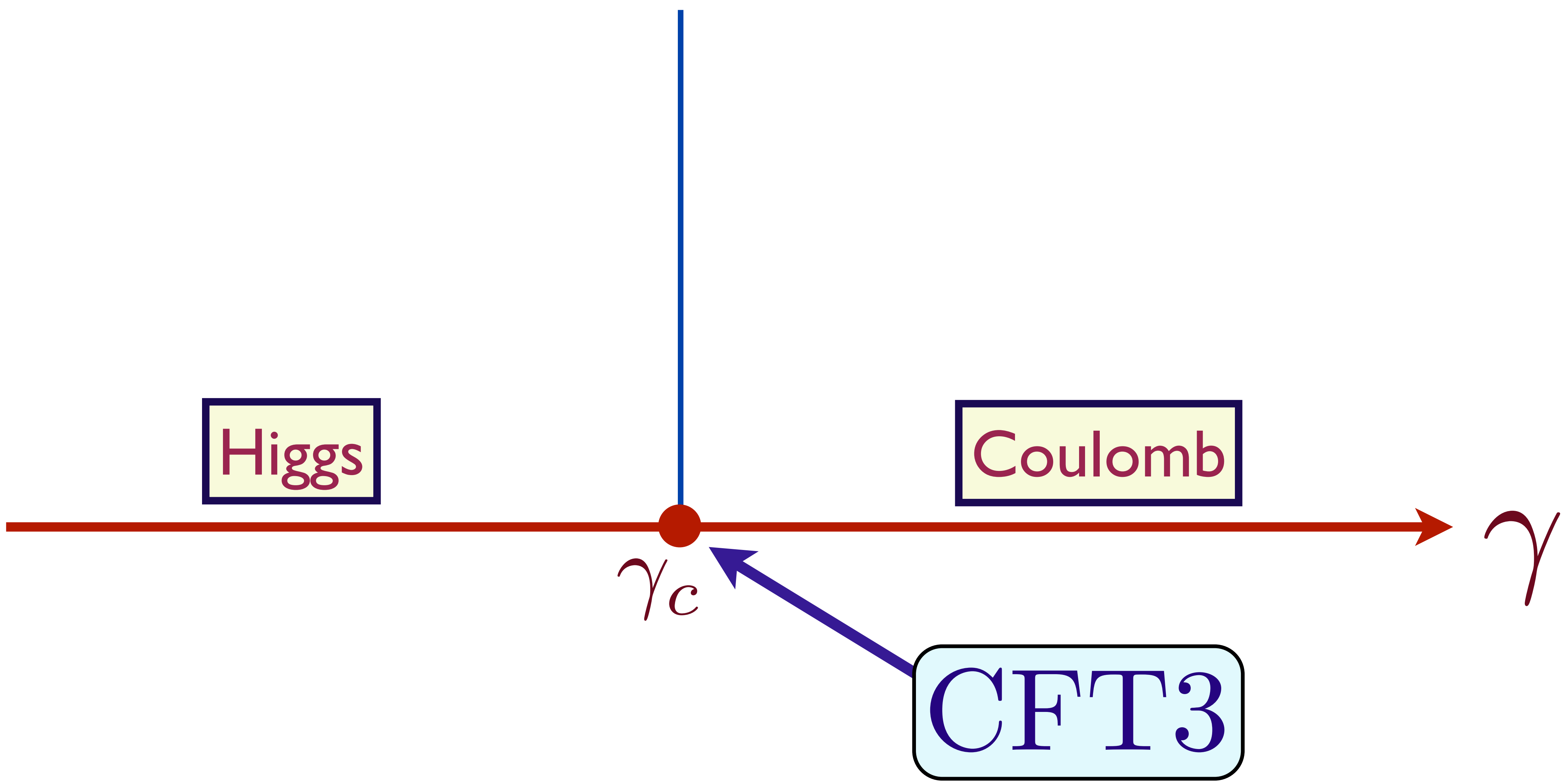
We obtain a U(1) gauge theory of

- bosonic neutral spinons z_α;
- spinless, charged fermions $\psi_\pm$ with small 'pocket' Fermi surfaces;
- an emergent U(1) gauge field A_μ.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Phys. Rev. B* **80**, 155129 (2009).

- Begin with a CFT3: the CP^1 model.

$$\mathcal{L}_z = \frac{1}{\gamma}\left|(\partial_\mu - iA_\mu)z_\alpha\right|^2 \quad ; \quad |z_\alpha|^2 = 1$$



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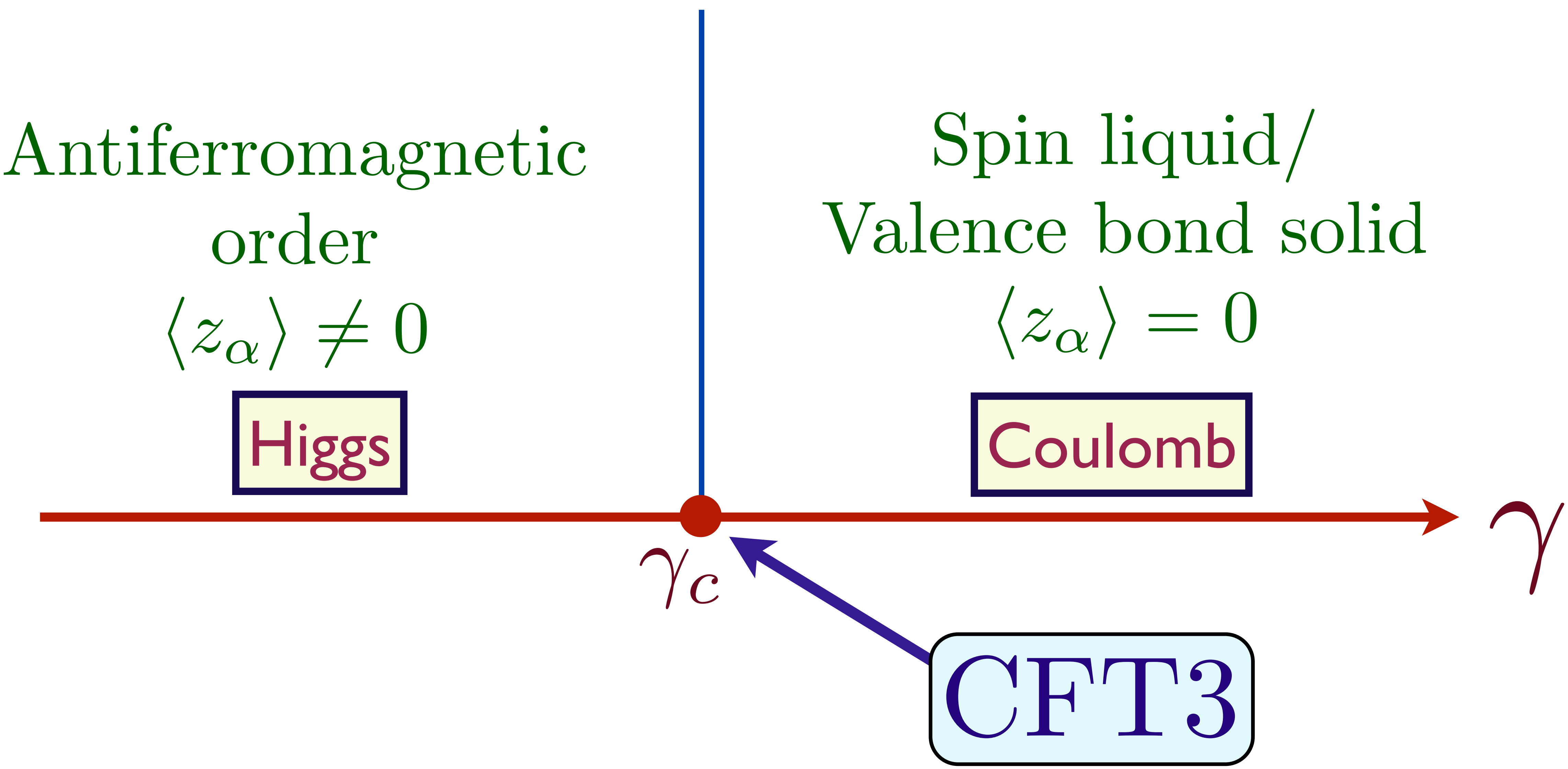
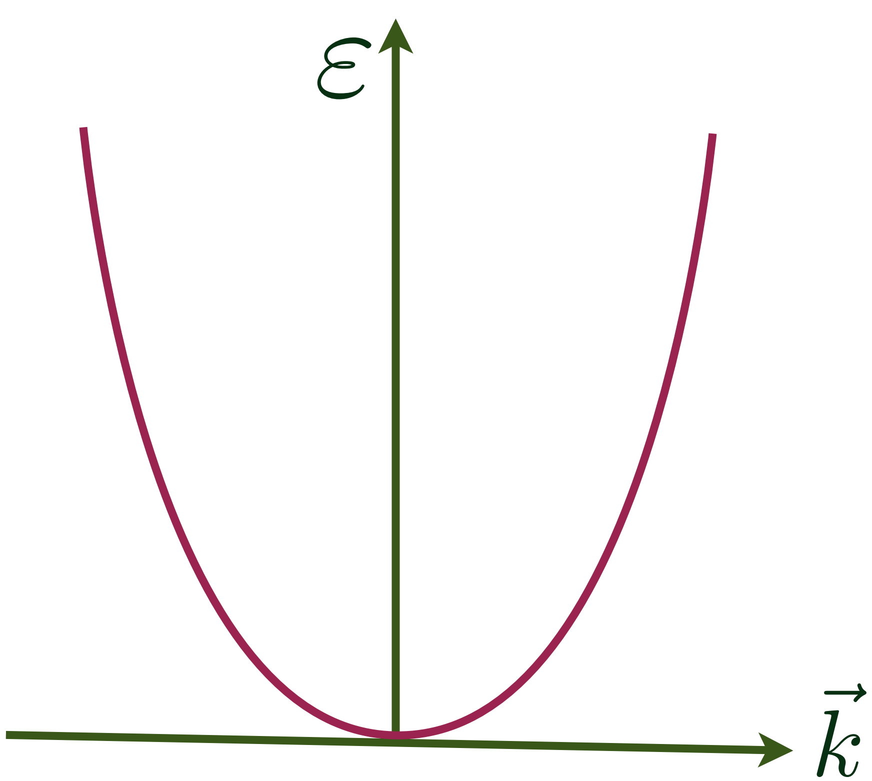
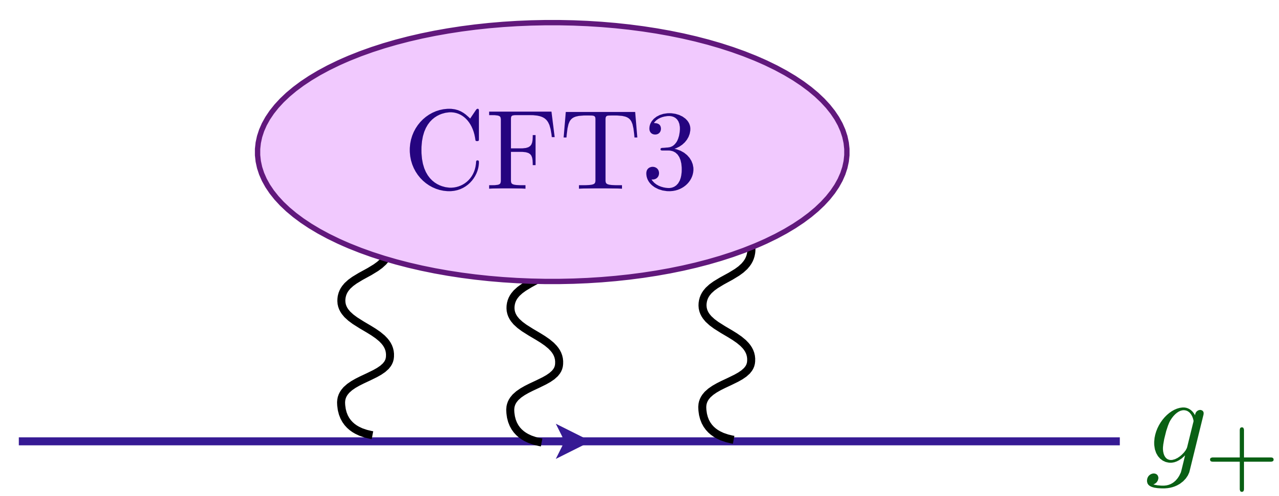


- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges

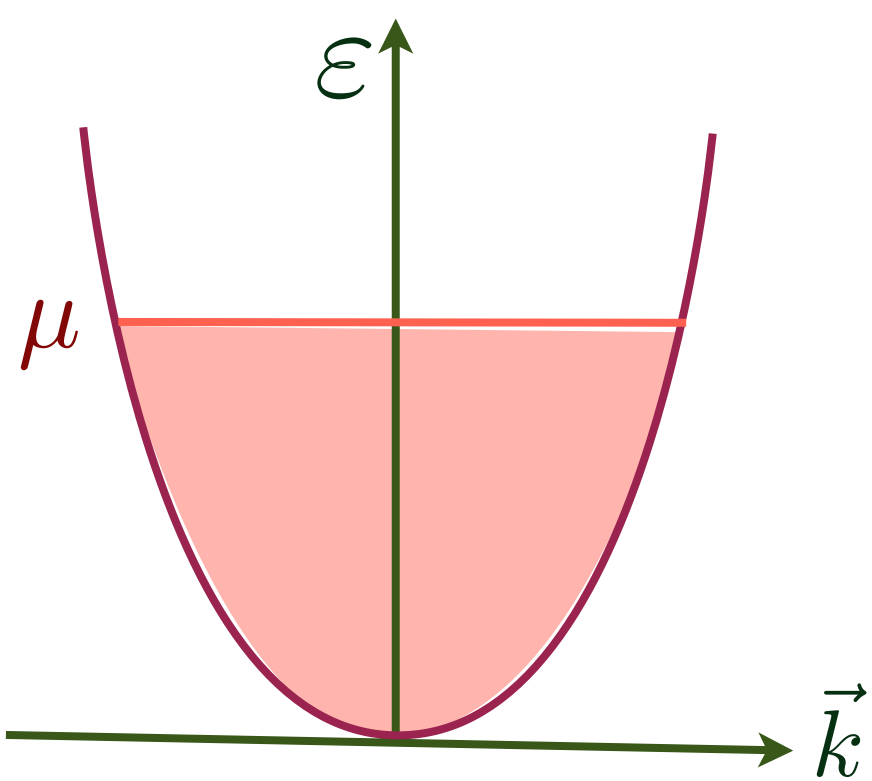
$$\mathcal{L}_f = g_+^\dagger \left(\frac{\partial}{\partial \tau} - iA_\tau - \frac{1}{2m}\left(\vec{\nabla} - i\vec{A}\right)^2 \right) g_+ + g_-^\dagger \left(\frac{\partial}{\partial \tau} + iA_\tau - \frac{1}{2m}\left(\vec{\nabla} + i\vec{A}\right)^2 \right) g_-$$

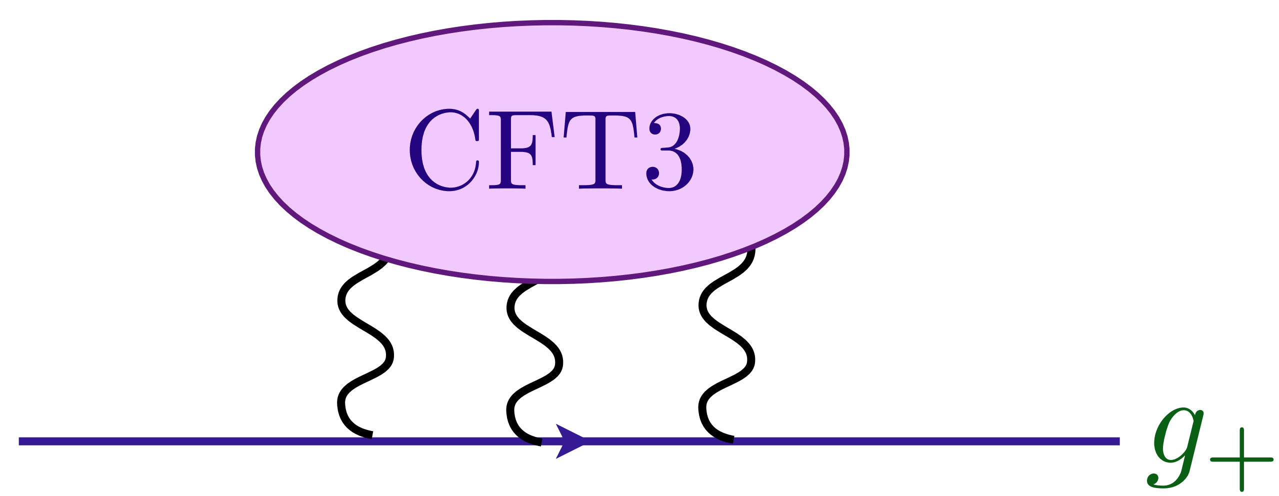




- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential:

$$\mathcal{L}_f = g_+^\dagger \left(\frac{\partial}{\partial \tau} - iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} - i\vec{A}\right)^2 \right) g_+ + g_-^\dagger \left(\frac{\partial}{\partial \tau} + iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} + i\vec{A}\right)^2 \right) g_-$$





Complete theory

$$\mathcal{L} = \mathcal{L}_z + \mathcal{L}_f$$

$$\mathcal{L}_z = \frac{1}{\gamma}\left|(\partial_\mu - iA_\mu)z_\alpha\right|^2 \quad ; \quad |z_\alpha|^2 = 1$$

$$\begin{aligned}\mathcal{L}_f \quad = \quad & g_+^\dagger \left(\frac{\partial}{\partial \tau} - iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} - i\vec{A}\right)^2 \right) g_+ \\ & + g_-^\dagger \left(\frac{\partial}{\partial \tau} + iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} + i\vec{A}\right)^2 \right) g_-\end{aligned}$$

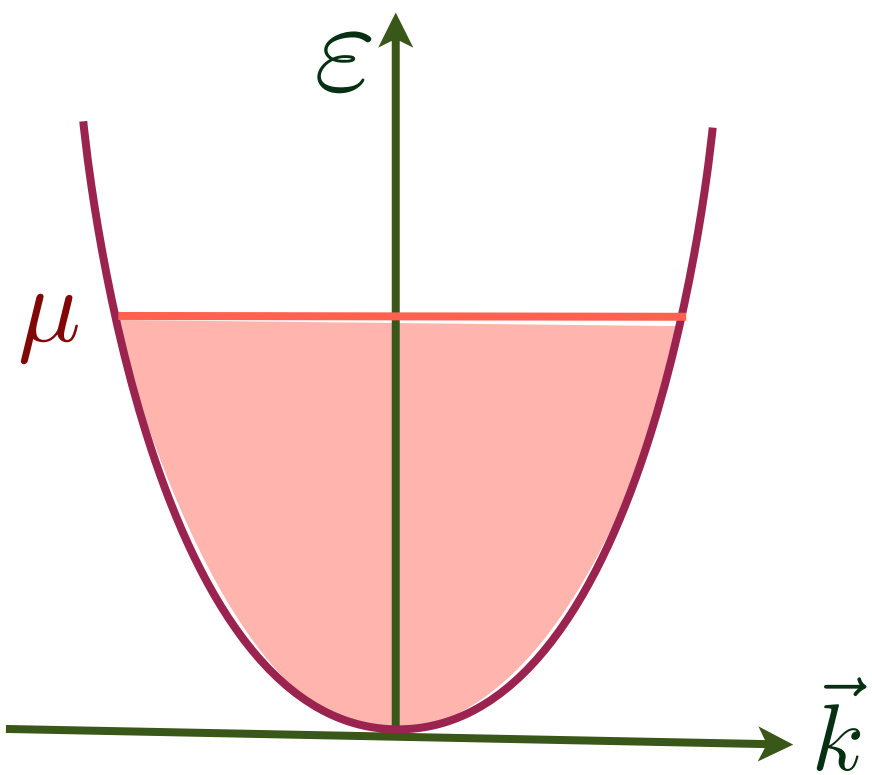
V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

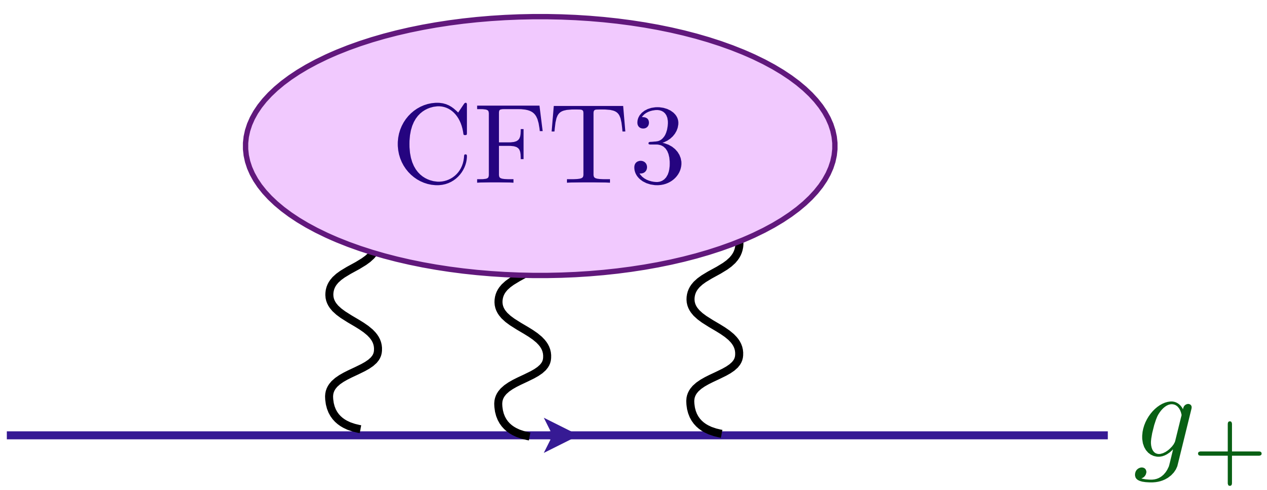
Theory has *many* similarities to holographic superconductors (Gubser, Hartnoll, Herzog, Horowitz) solved via the AdS/CFT correspondence, which (presumably) describe SYM3 theories in which gluinos pair via exchange of gluons into color singlets, and then Bose condense:

- Fermi surfaces with non-Fermi singularities in spectral functions
- Cooper pairs which are gauge neutral
- Are obtained after doping a CFT3 with finite density of a conserved global charge
- Fermion and current spectral functions in superconducting and normal states have many similarities to cuprates

- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential:

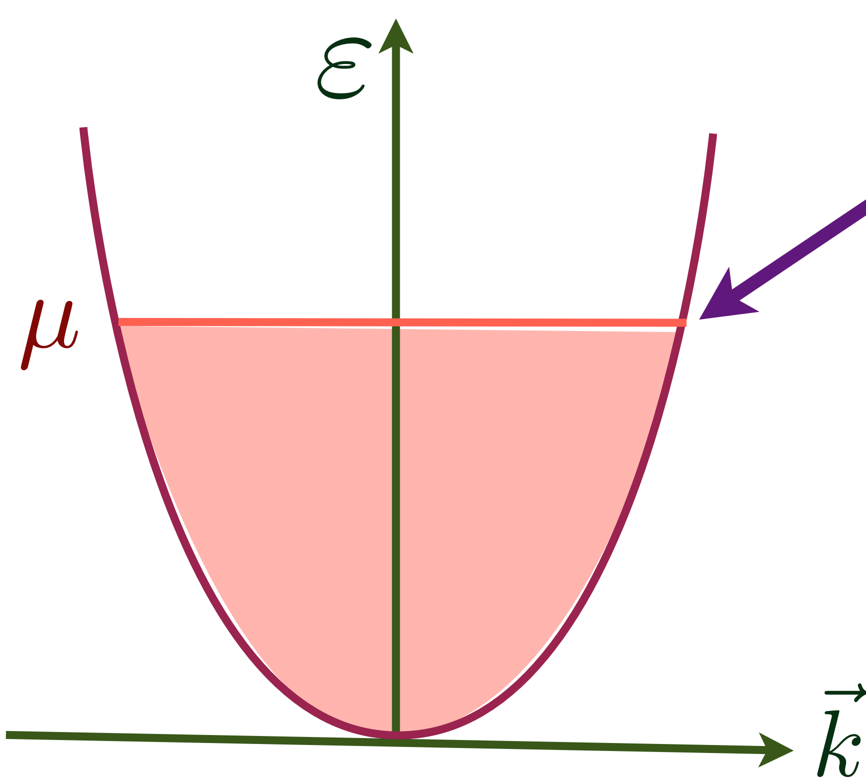
$$\mathcal{L}_f = g_+^\dagger \left(\frac{\partial}{\partial \tau} - iA_\tau - \mu - \frac{1}{2m} \left(\vec{\nabla} - i\vec{A} \right)^2 \right) g_+ + g_-^\dagger \left(\frac{\partial}{\partial \tau} + iA_\tau - \mu - \frac{1}{2m} \left(\vec{\nabla} + i\vec{A} \right)^2 \right) g_-$$

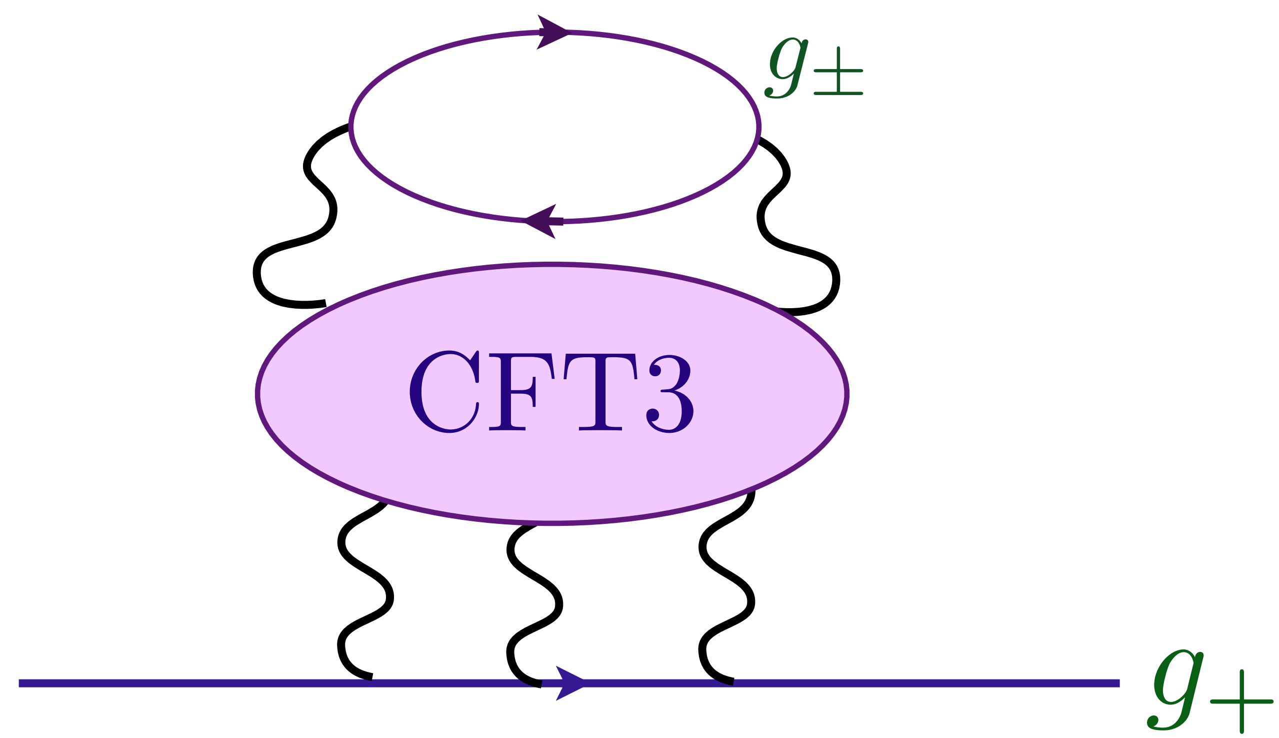




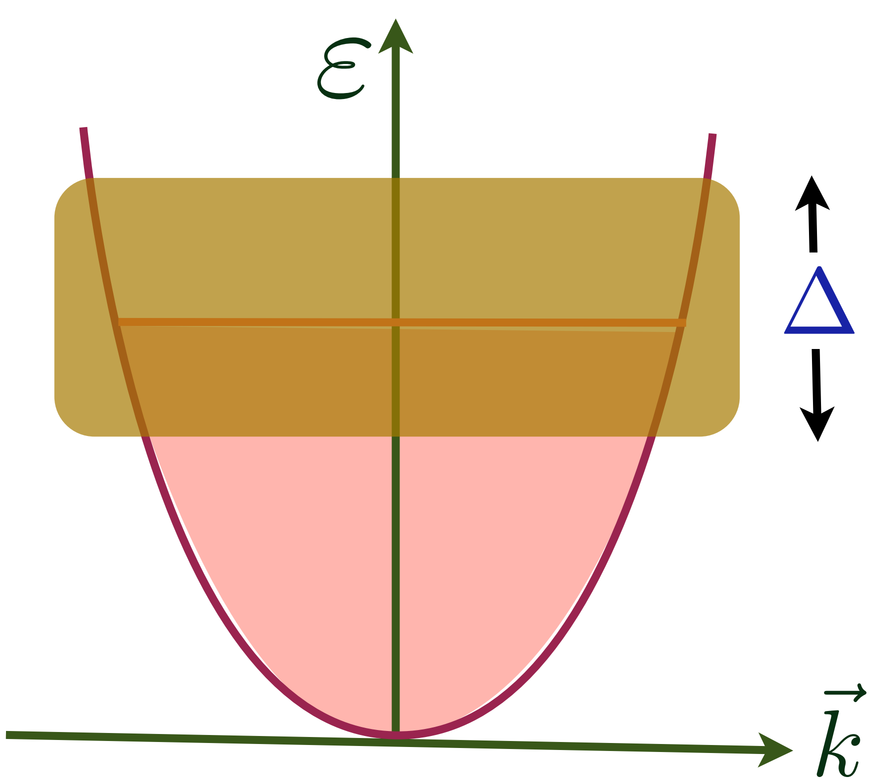
- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential: leads to a marginal Fermi liquid of $g_\pm$ (not electrons)

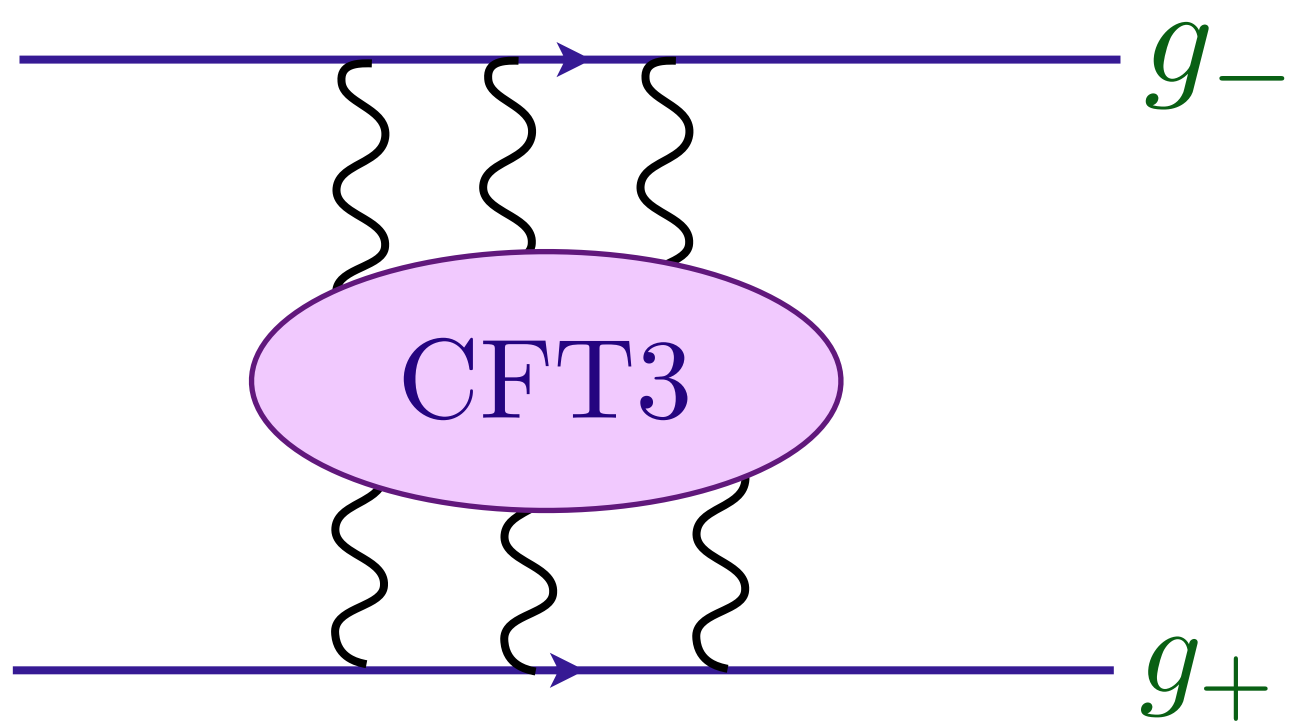
$$G(\vec{k},\omega) = \frac{1}{\omega - v_F(|\vec{k}| - k_F) + c\,\omega[\ln(|\omega|) + i\pi\mathrm{sgn}(\omega)]}$$





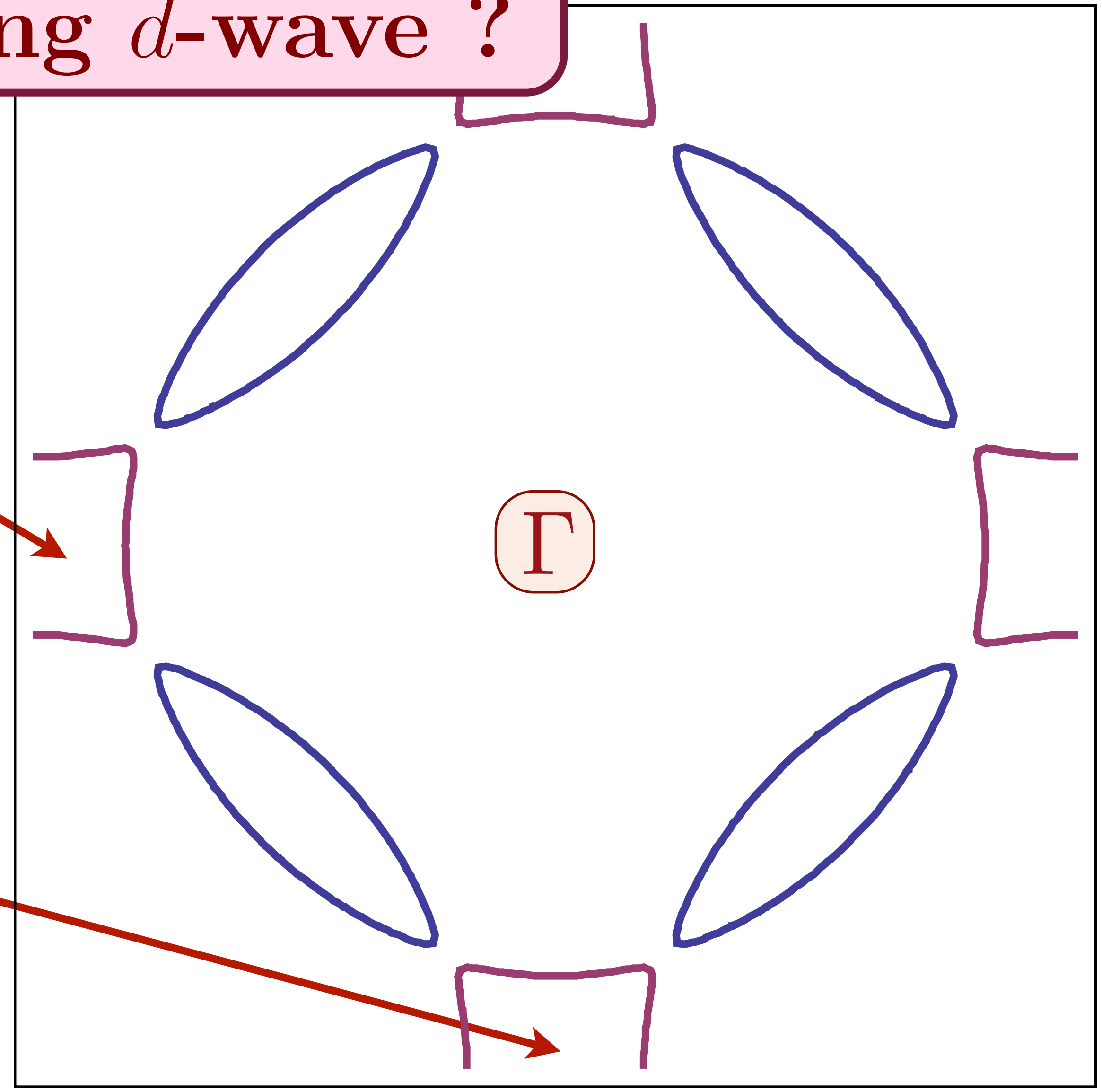
- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential: leads to a marginal Fermi liquid of $g_\pm$ (not electrons)
- Low T state is a superconductor with $\langle g_+ g_- \rangle = \Delta \neq 0$





Why is the pairing d-wave ?

Electron $c_{2\alpha}$,
spinless fermion $g_\pm$



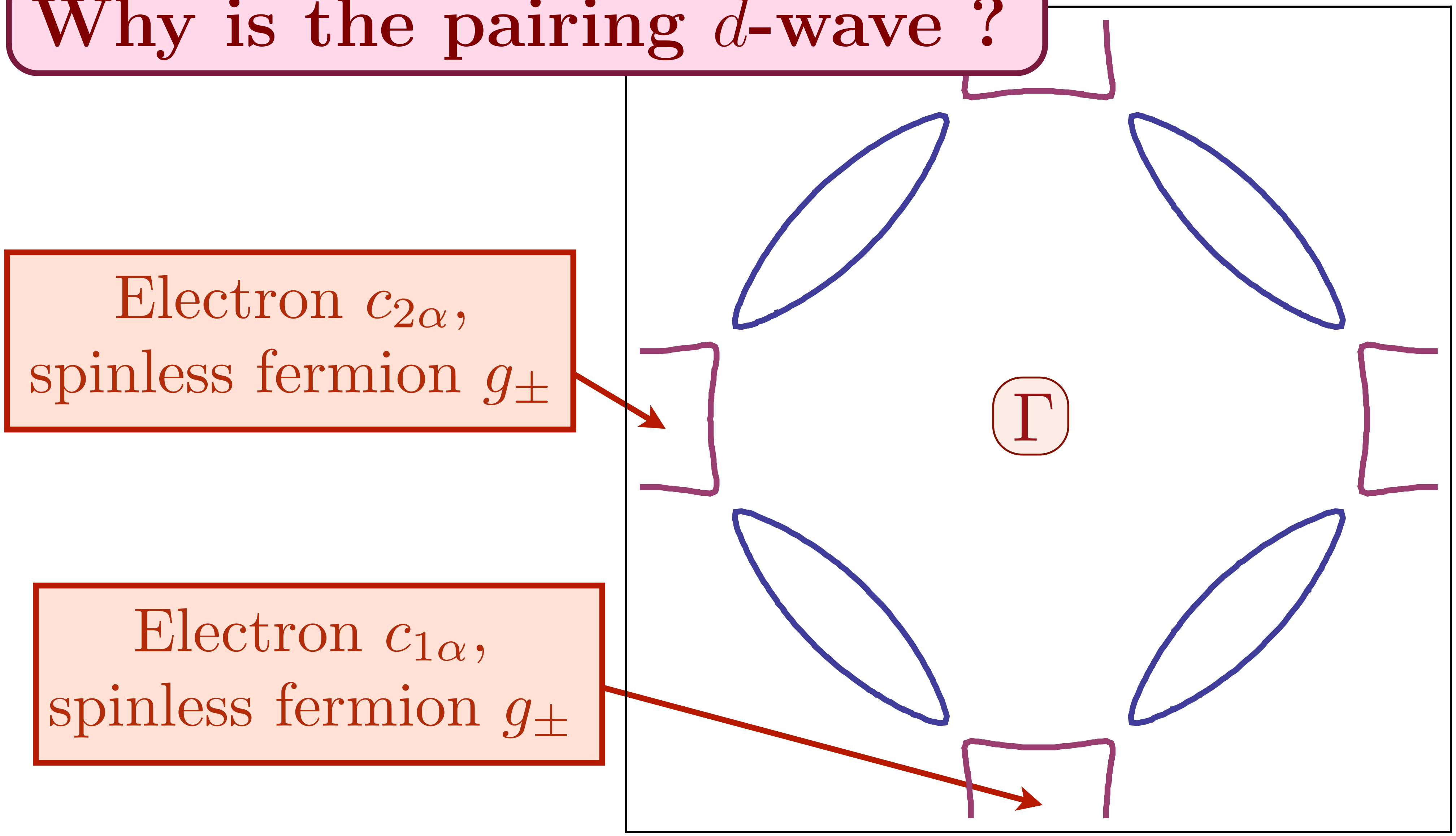
Electron $c_{1\alpha}$,
spinless fermion $g_\pm$

Focus on pairing near $(\pi, 0)$, $(0, \pi)$, where $\psi_\pm \equiv g_\pm$, and the electron operators are

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

$$\mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

Why is the pairing d-wave ?



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Why is the pairing d-wave ?

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Attractive gauge forces lead to simple s-wave pairing of the $g_\pm$

$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

$i.e.$ d-wave pairing !

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

T=0 Phase diagram

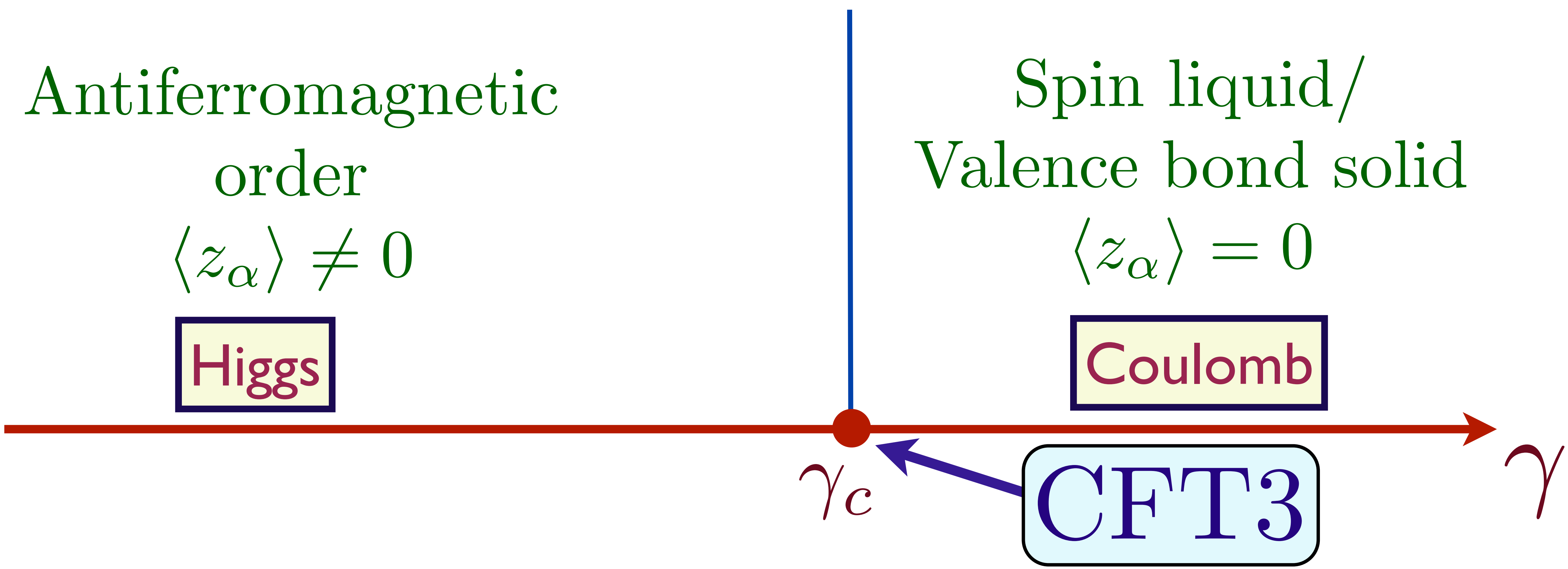
$$\mathcal{L}_z = \frac{1}{\gamma} |(\partial_\mu - i A_\mu) z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$

Antiferromagnetic order

$\langle z_\alpha \rangle \neq 0$

Spin liquid/ Valence bond solid

$\langle z_\alpha \rangle = 0$



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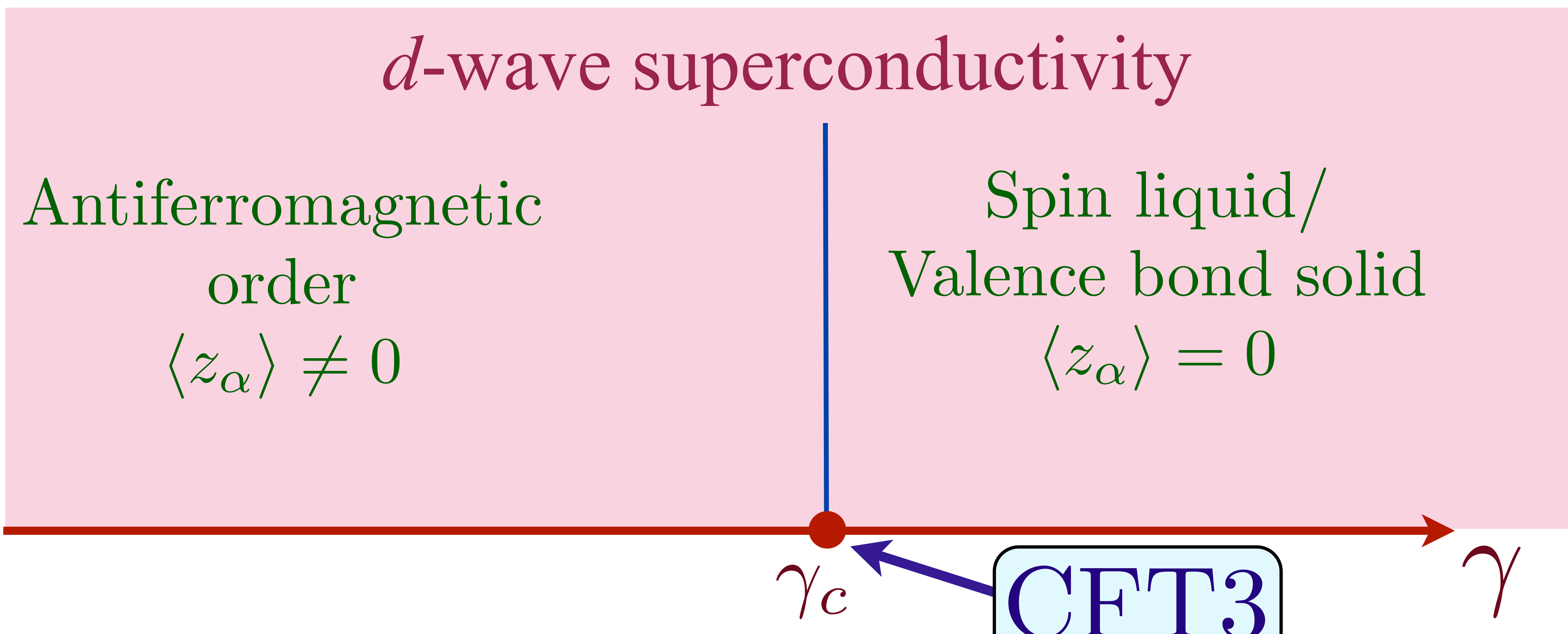

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

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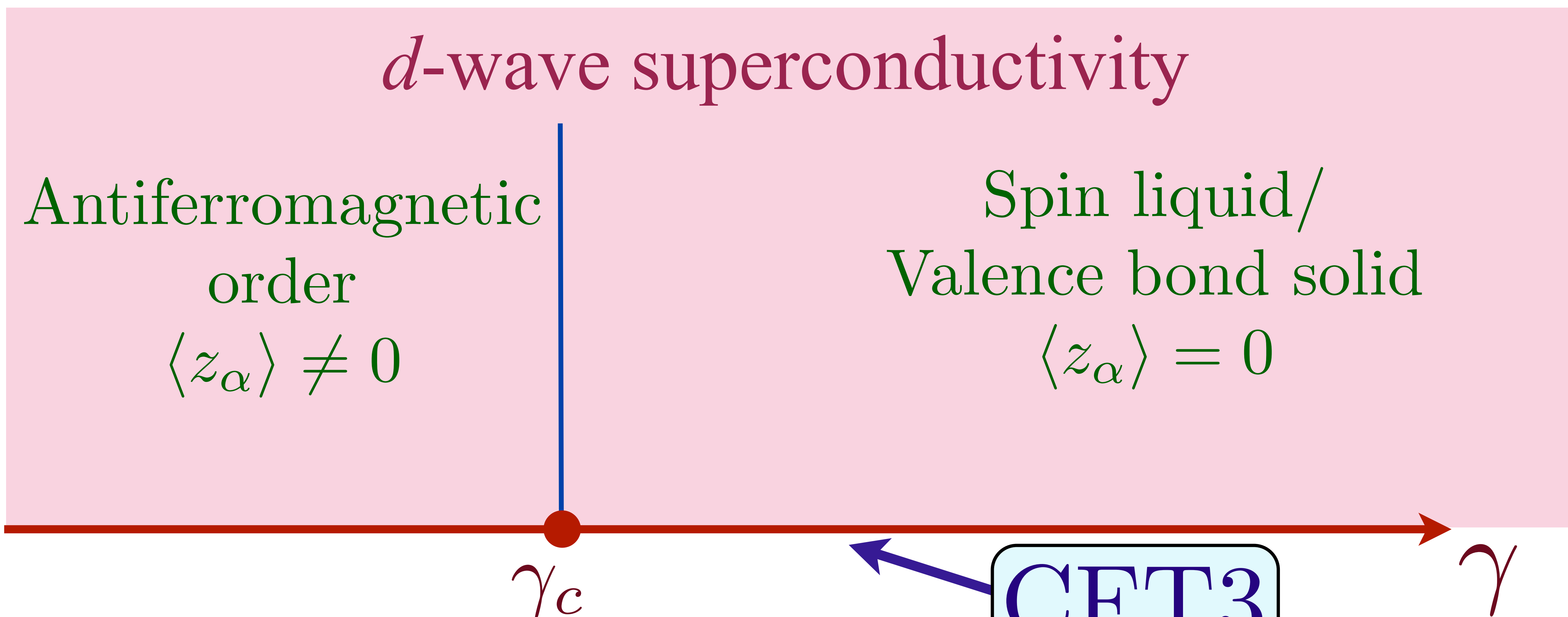
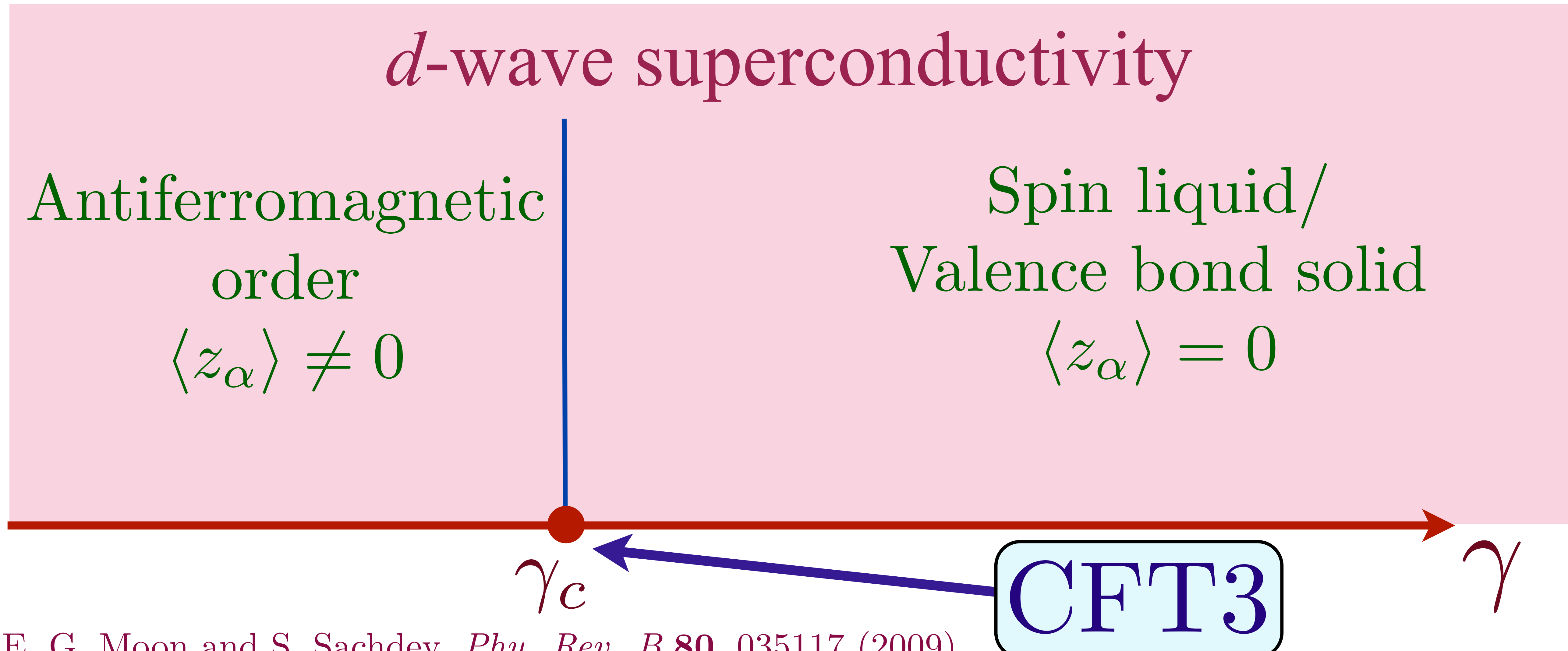


E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

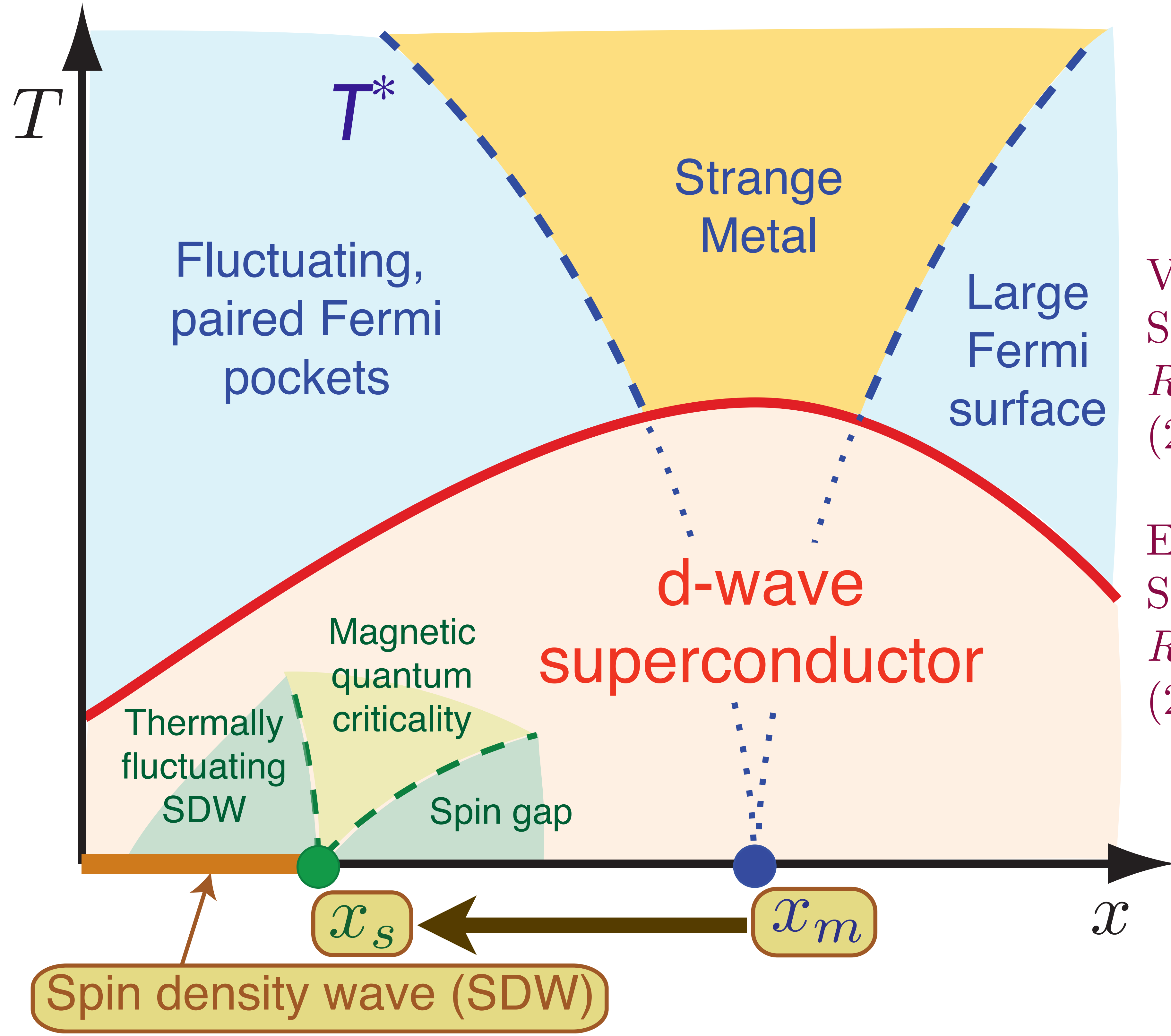
T=0 Phase diagram

Competition between antiferromagnetism and superconductivity shrinks region of antiferromagnetic order: feedback of "probe fermions" on CFT is important



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Theory of quantum criticality in the cuprates

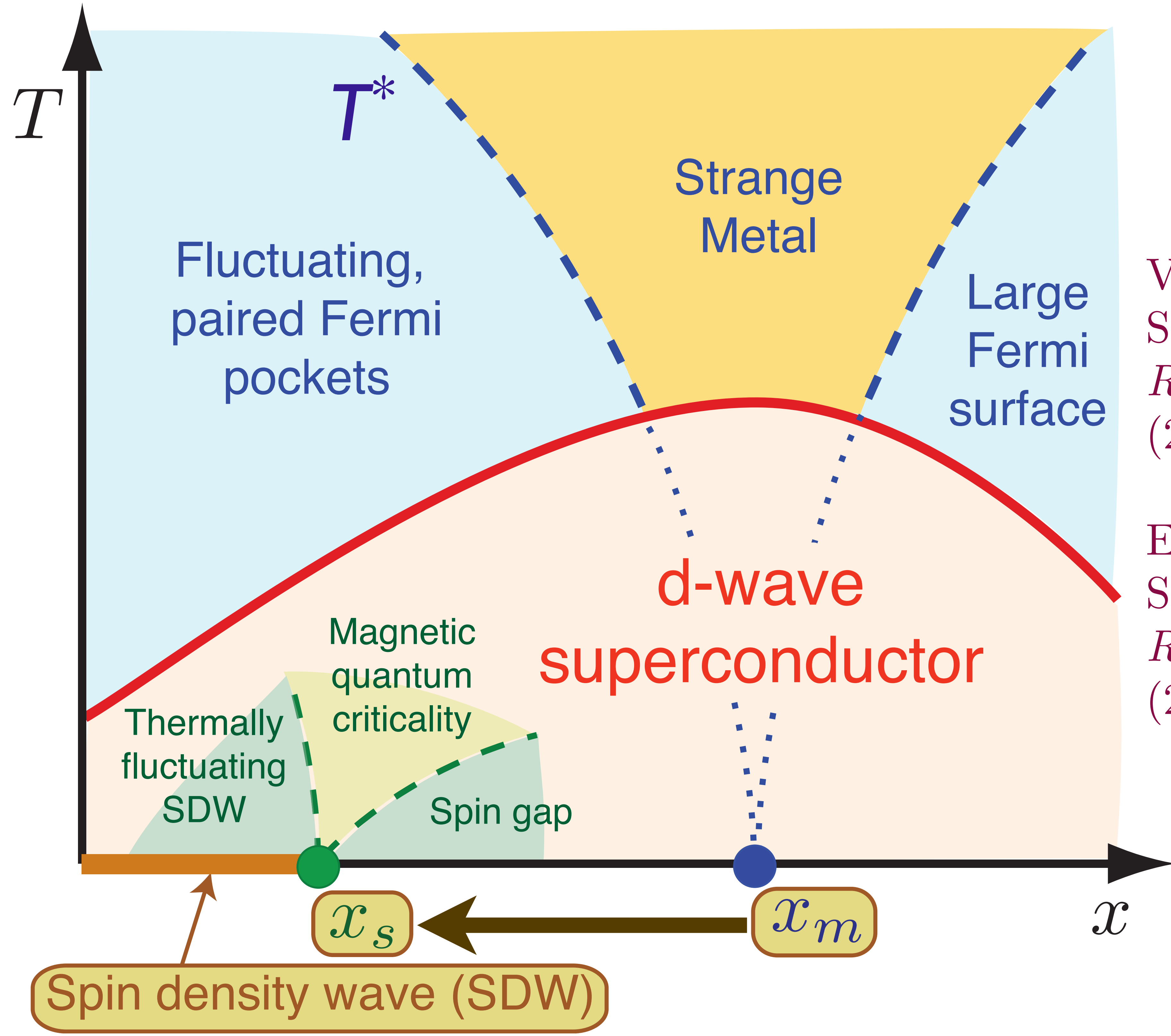


V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

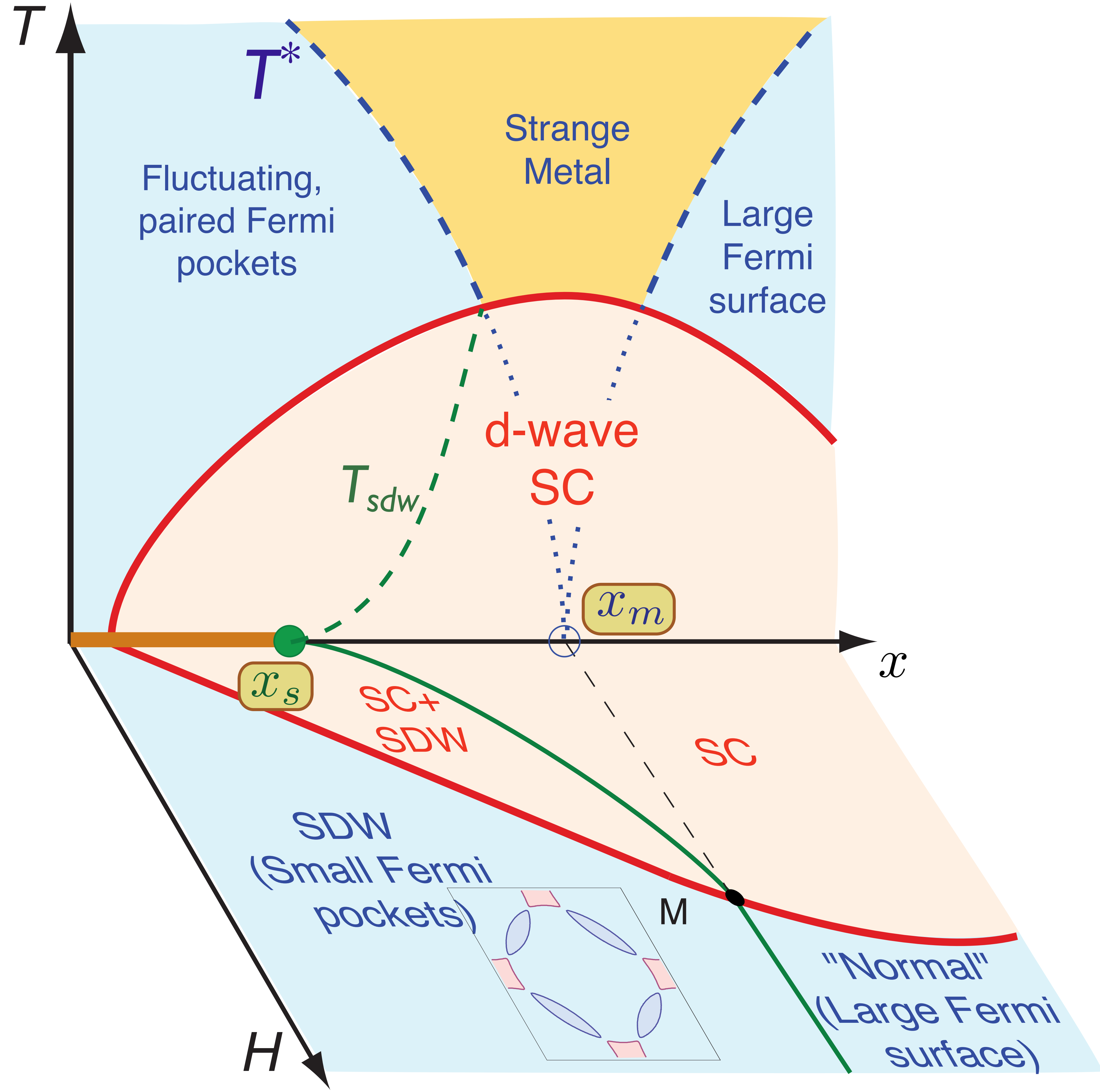
Theory of quantum criticality in the cuprates



V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

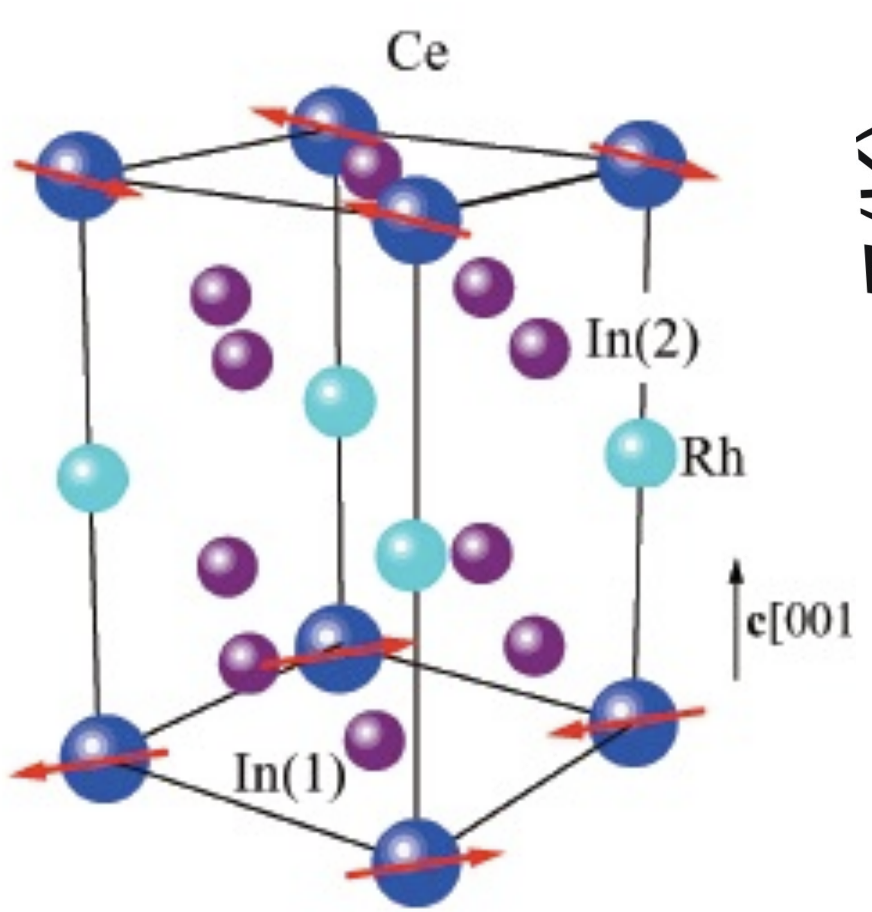
Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.

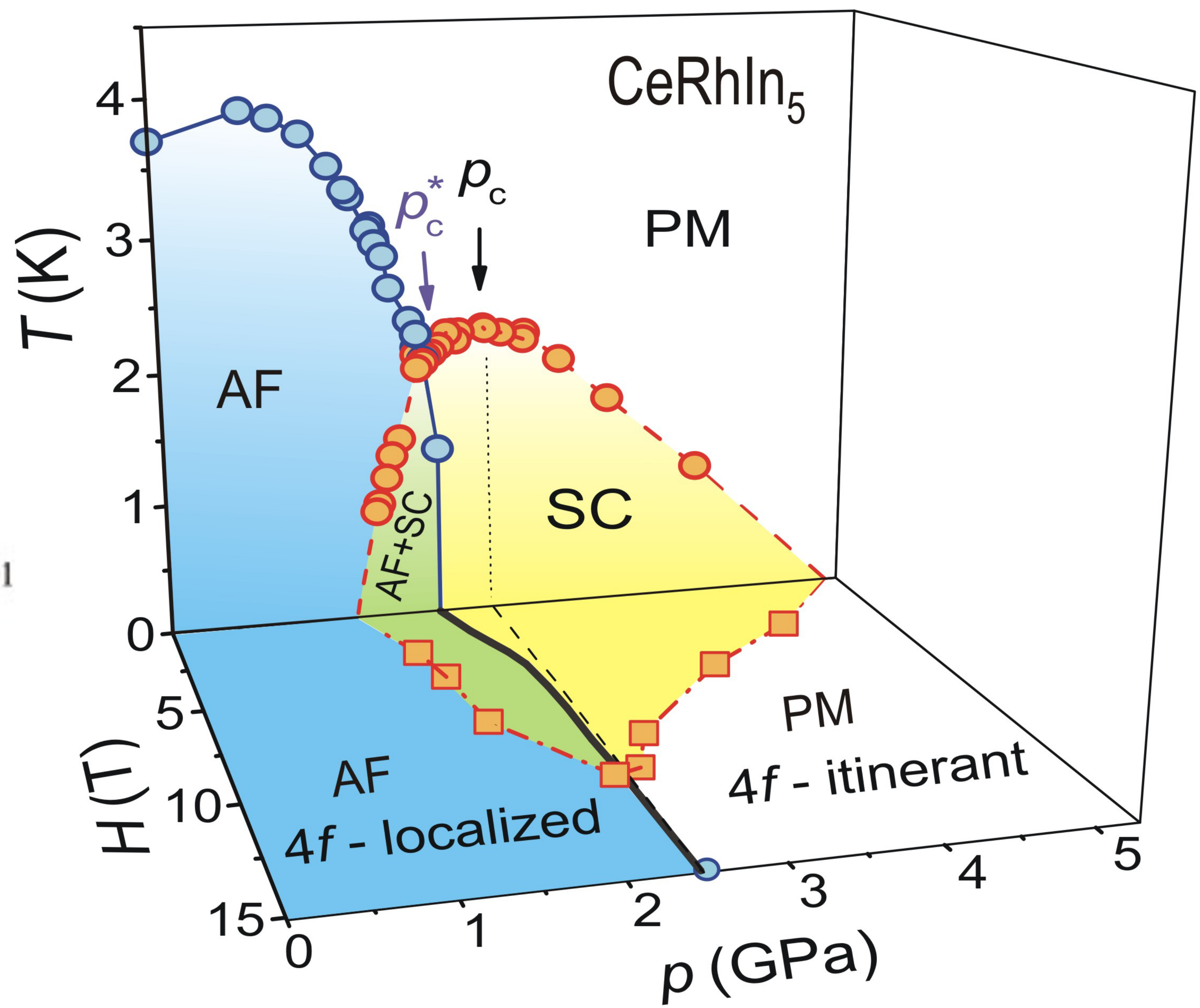


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

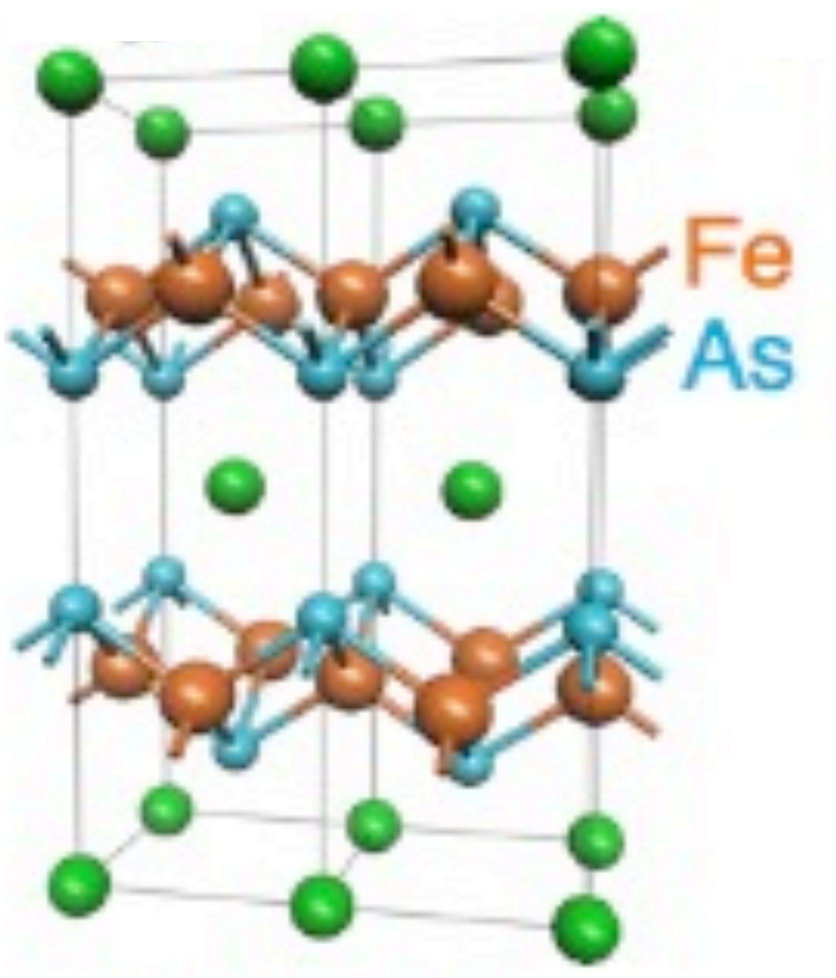
Similar phase diagram for $CeRhIn_5$



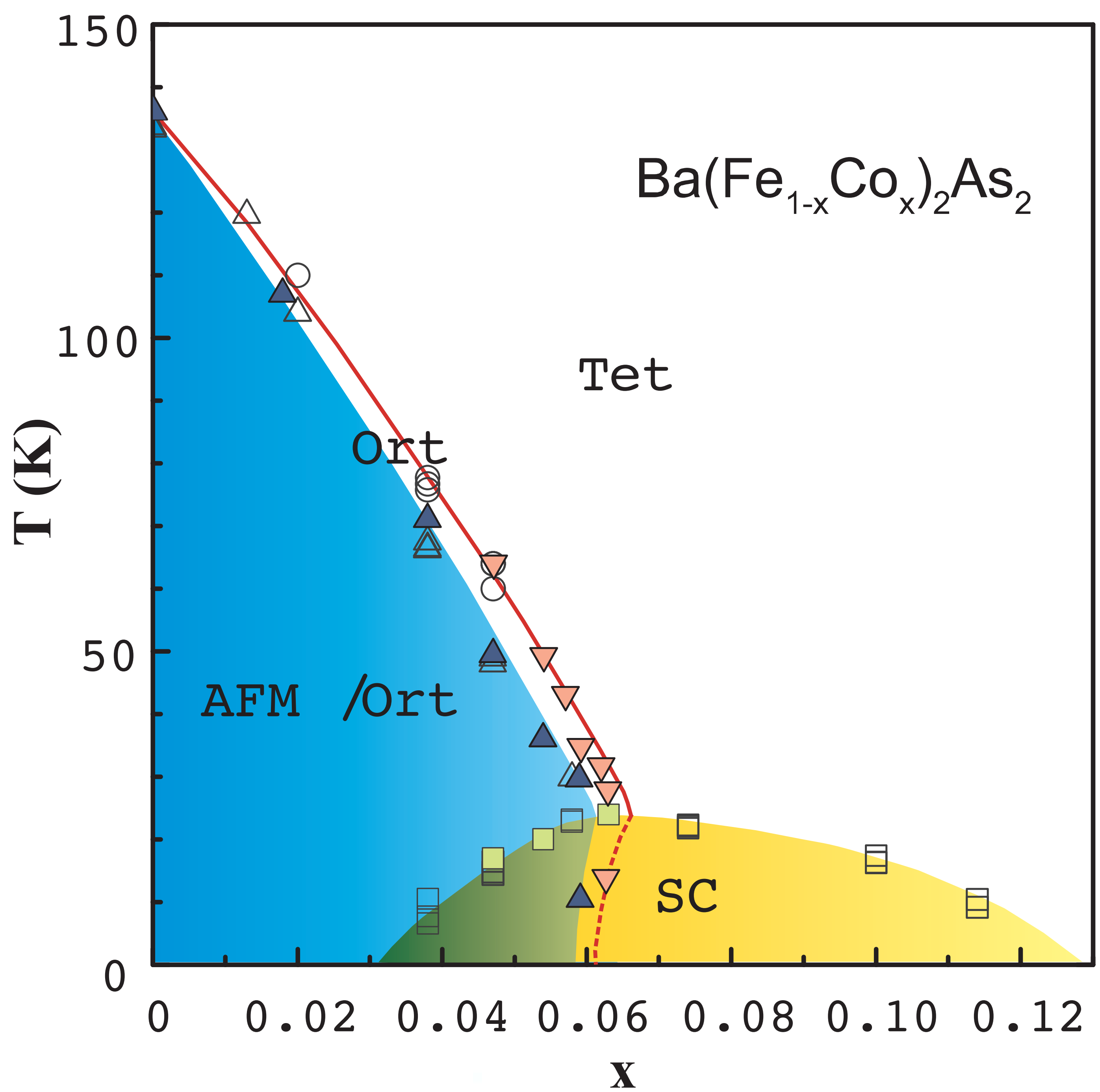


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Similar phase diagram for the pnictides



BaFe$_2$As$_2$
Unit cell



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, arXiv:0911.3136.

Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density

Conclusions

Gauge theory for pairing of Fermi pockets in a metal with fluctuating spin density wave order: Many qualitative similarities to holographic strange metals and superconductors